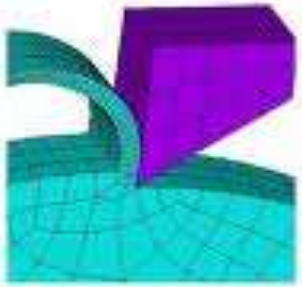
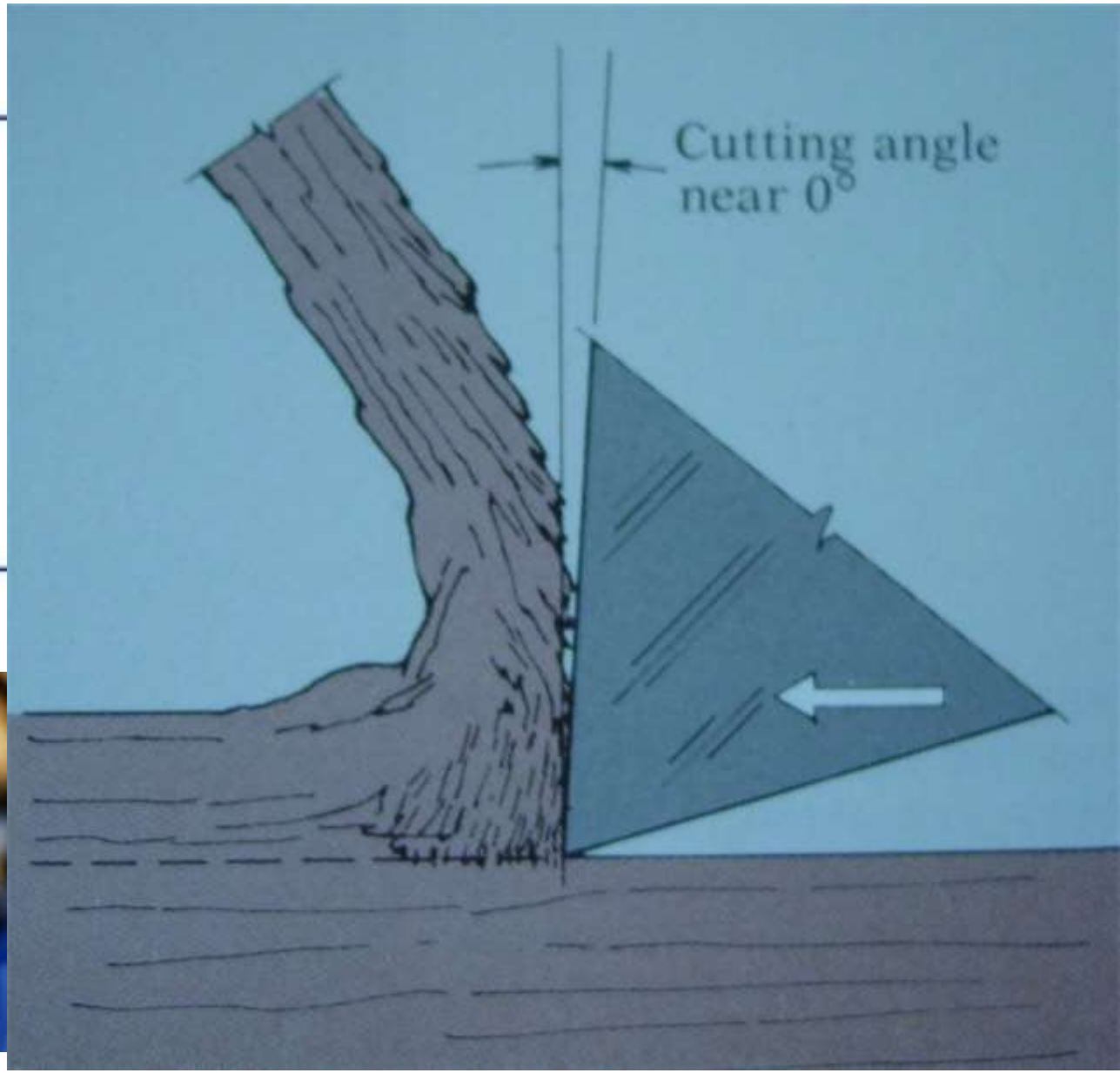
THEORY OF METAL MACHINING





Machining variables

- There are two kinds of machining variables:

1. Independent variables
2. Dependent variables.

- In cutting process, the major ***independent variables***, that is, the variables that we can change and control directly, are :

1. Type of cutting tool and its condition (the tool material)
2. Tool shape and its sharpness
3. work part material and its conditions
4. Cutting conditions (speed, feed and depth of cut)
5. Type of cutting fluids
6. Characteristics of the machine tool, particularly its stiffness and damping
7. Tool holders and fixturing device

Machining variables

- The dependent variables, which are the variables that are influenced by changes in the independent variables, are:

1. Type of chip produced
2. Force required and energy dissipated in the cutting process
3. The temperature rise in the work piece, the chip and the tool
4. Surface finish and integrity of the work piece after it is machined

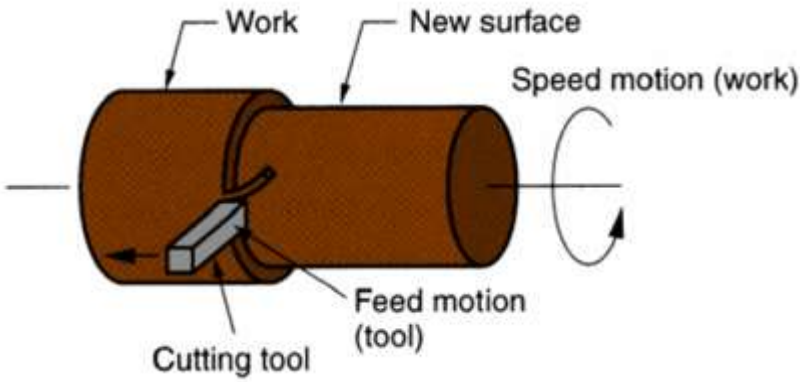
- It is important to study the complex interrelationships among these variables. Why???????

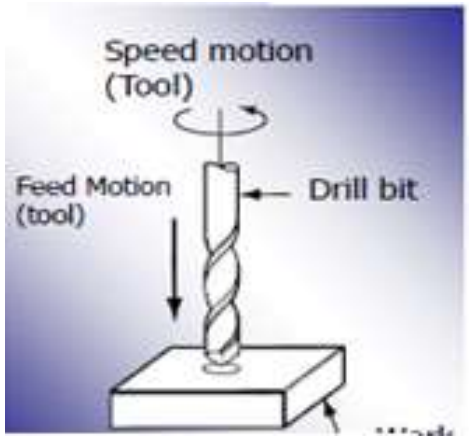
Machining variables

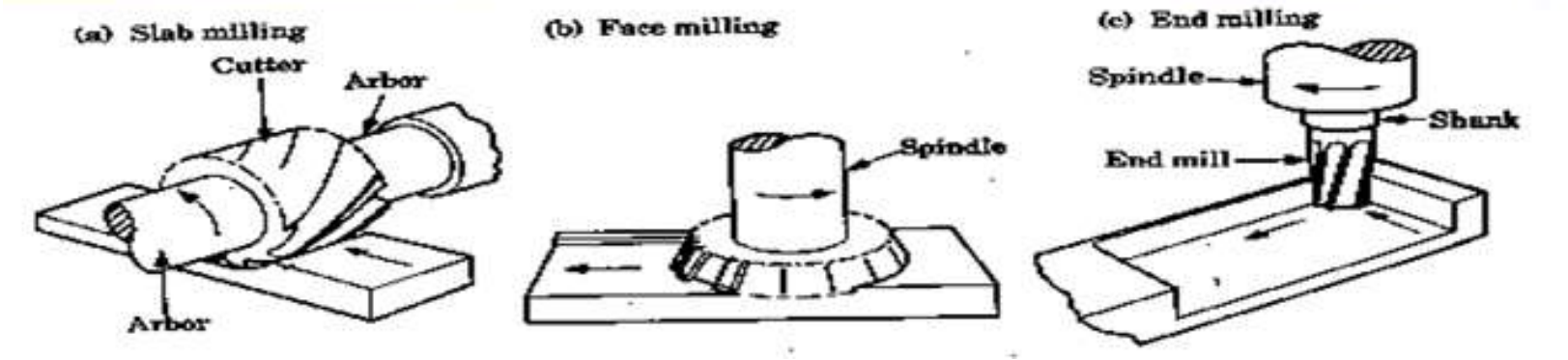
1. ***If the surface finish of workpiece being cut is poor and unacceptable***, **we need to know which of the pervious independent variables do we modify?**
2. ***If the workpiece becomes too hot, thus affecting its properties,*** **what modifications should be made?**
3. ***If the cutting tool wears rapidly and becomes dull***, **what should we do?** Do we change the cutting speed, the depth of cut, the tool material itself, or some other variables?
4. ***If the dimensional tolerance of the machined workpiece is over the specified limits***, **what modification should be made?**
5. If the cutting tool begins to vibrate and chatter, thus affecting surface finish, so **what should be done to eliminate this problem?**

Mechanism of chip formation

- Machining processes remove material from the workpiece surface by producing chips as shown the below figure.

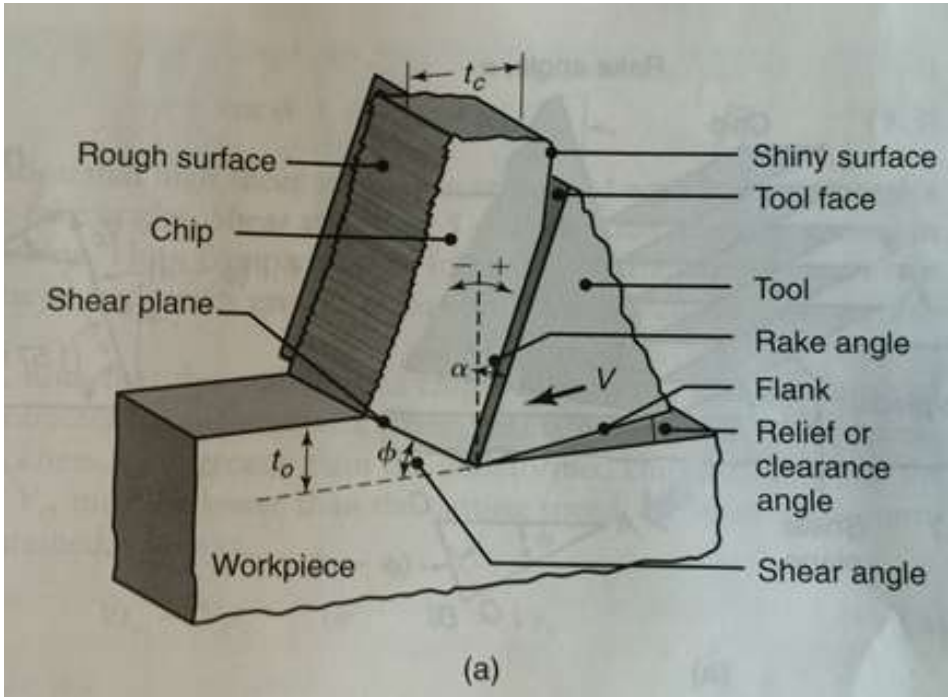


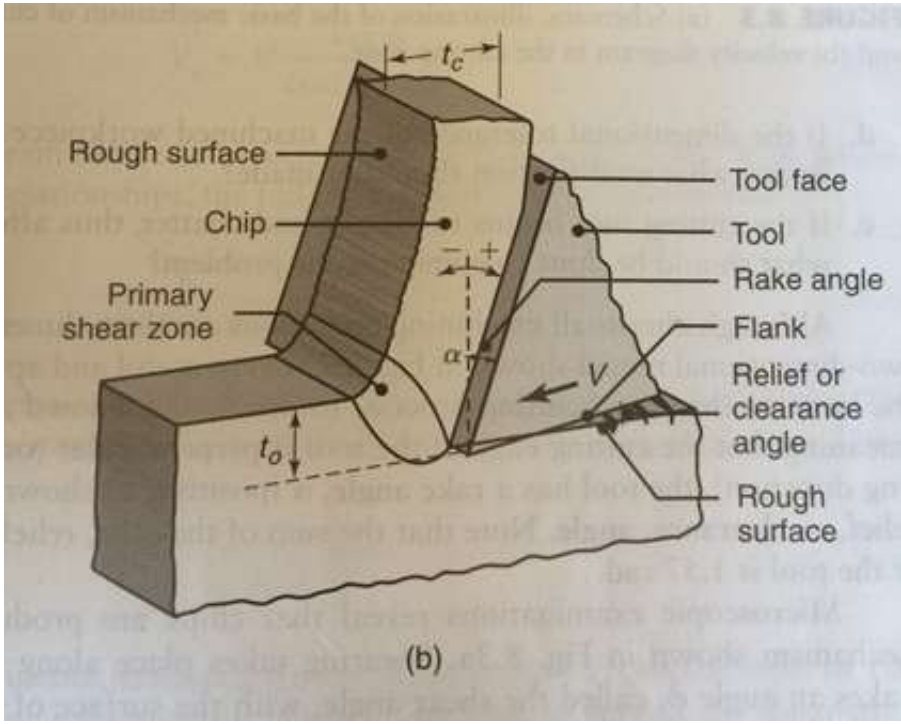




Mechanism of chip formation

- The basic mechanics of chip formation, which is essentially the same for all cutting operations is represented by two-dimensional model





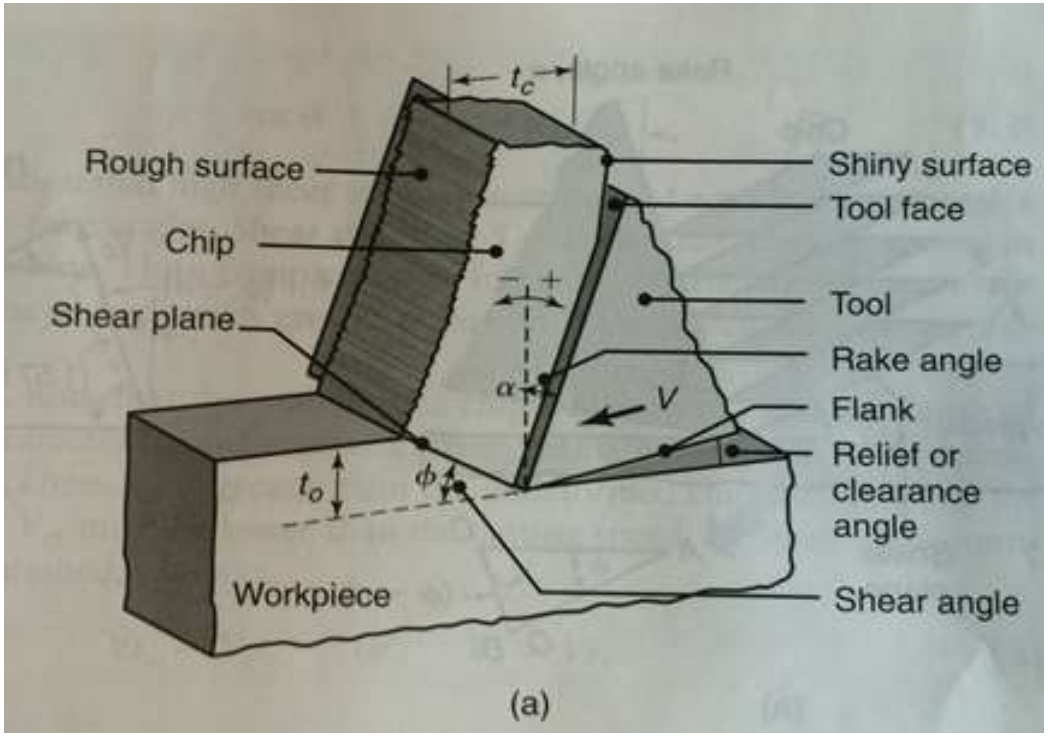
(a) orthogonal cutting with well-defined shear plane also known as merchant

(b) orthogonal cutting without a well-defined shear plane

Mechanism of chip formation

- ***In this model***

1. A tool moves along the workpiece at a certain velocity (cutting speed), V, and a depth of cut, t_o (in orthogonal model)
2. A chip is produced just a head of the tool by shearing the material continuously along the shear plane

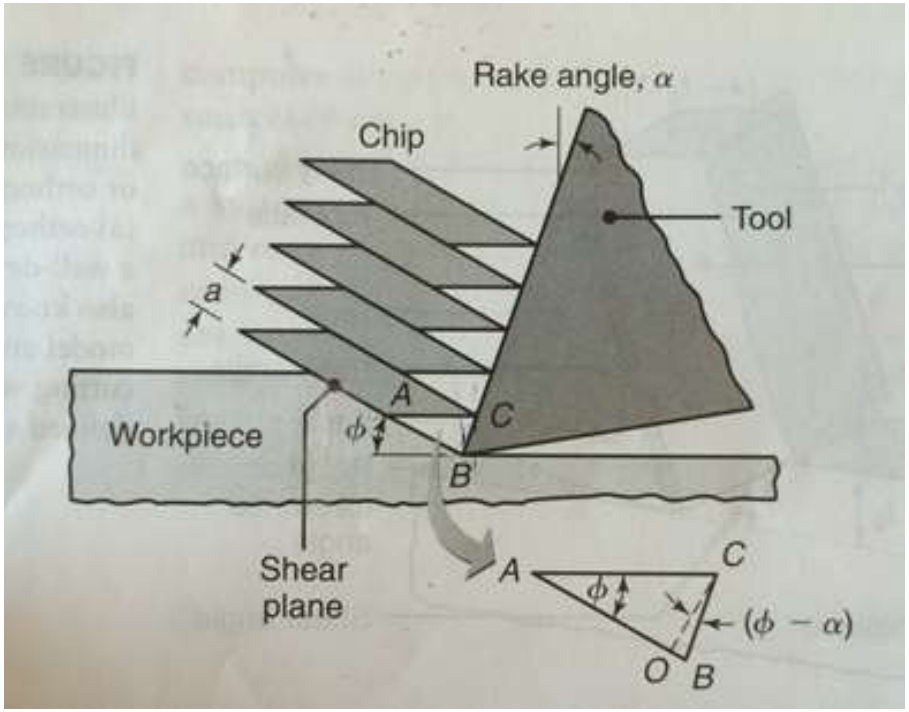


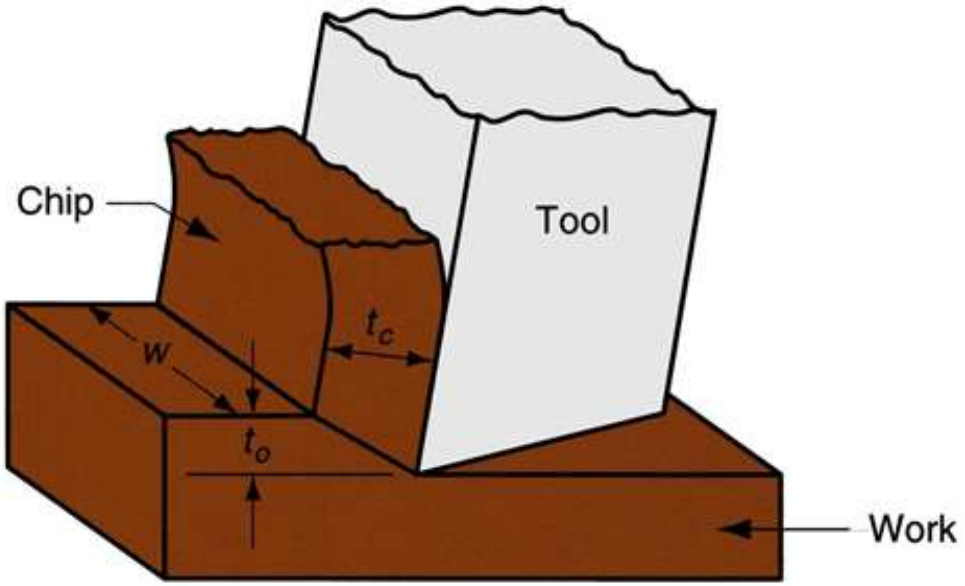
Mechanism of chip formation

- In this model, which is known a orthogonal cutting [meaning that the cutting edge of the tool is perpendicular (orthogonal) to the cutting direction]
- The tool has

1. a rake angle (α)
2. And a relief or clearance angle

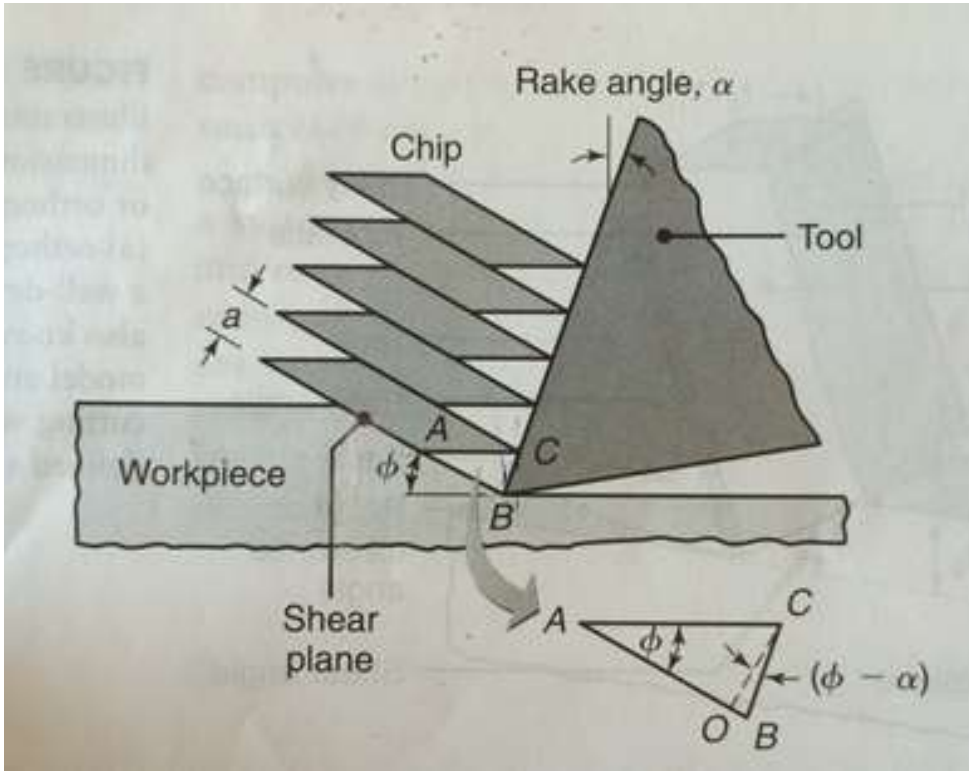
- The sum of the rake, relief and included angles of the tool is 1.57 rad (90^{o})





Mechanism of chip formation

- Microscopic examinations reveal that chips are produced by shearing mechanism as shown.
- Shearing take place along a shear plane, which make an angle Φ, called ***shear angle***, with the surface of the workpiece.
- Below the shear plane, the work piece is undeformed and above the shear plane, a chip is already formed and is moving up the face of the tool as cutting progress.
- Because of the relative movement, there is a friction between the chip and rake face of the tool



Cutting ratio

- Note that the thickness of the chip, t_c, can be determined if t_o, α, and Φ are known.
- The ratio of t_o to t_c is known as the cutting ratio, r, which can be expressed as:

$$r = \frac{t_o}{t_c}$$

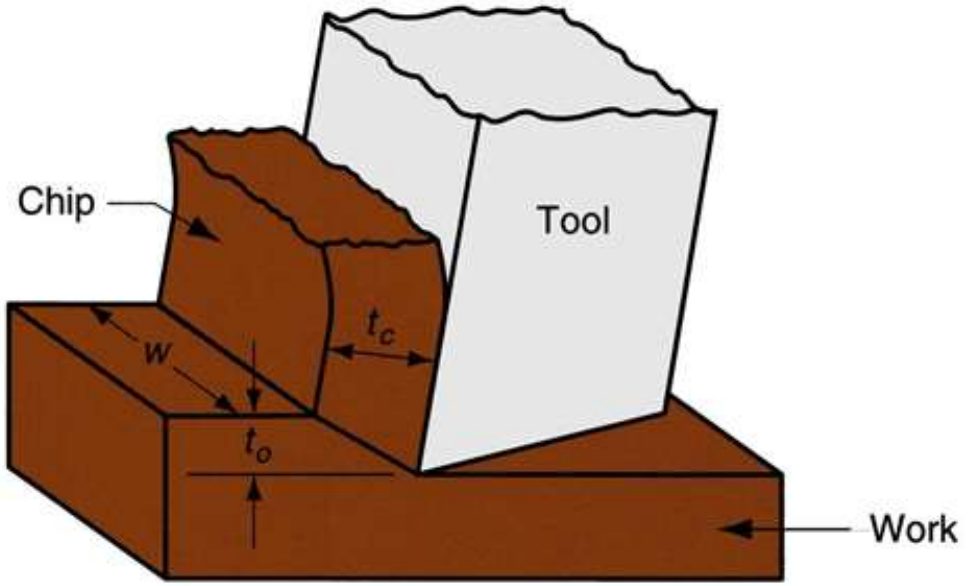
  - where:

  r = *chip thickness ratio*;

  t_o = thickness of the chip prior to chip formation;

  t_c = chip thickness after separation

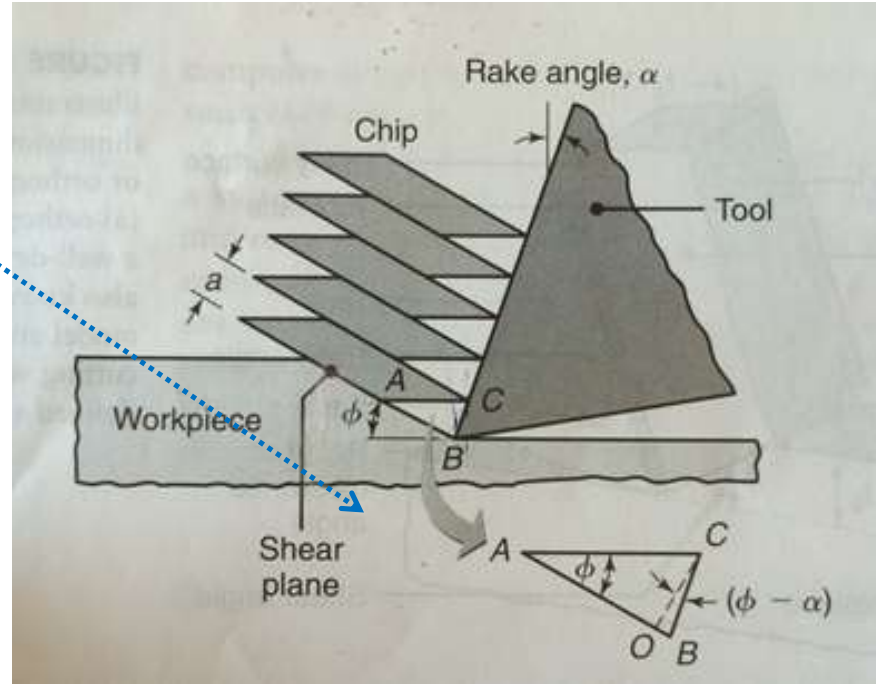
- Chip thickness after cut always greater than before, so chip ratio always less than 1.0



Cutting ratio

- Cutting relation Can be derived using trigonometric relations here.

$$r = \frac{\sin\phi}{\cos(\phi - \alpha)}$$



- By rearranging the above equation based on the geometric parameters of the orthogonal model, and using the trigonometric identity
  cos(φ - α) = cos φ cos α + sin φ sin α
  the shear plane angle ϕ can be determined as:

$$\tan\phi = \frac{r\cos\alpha}{1 - r\sin\alpha}$$

where

r = chip ratio,

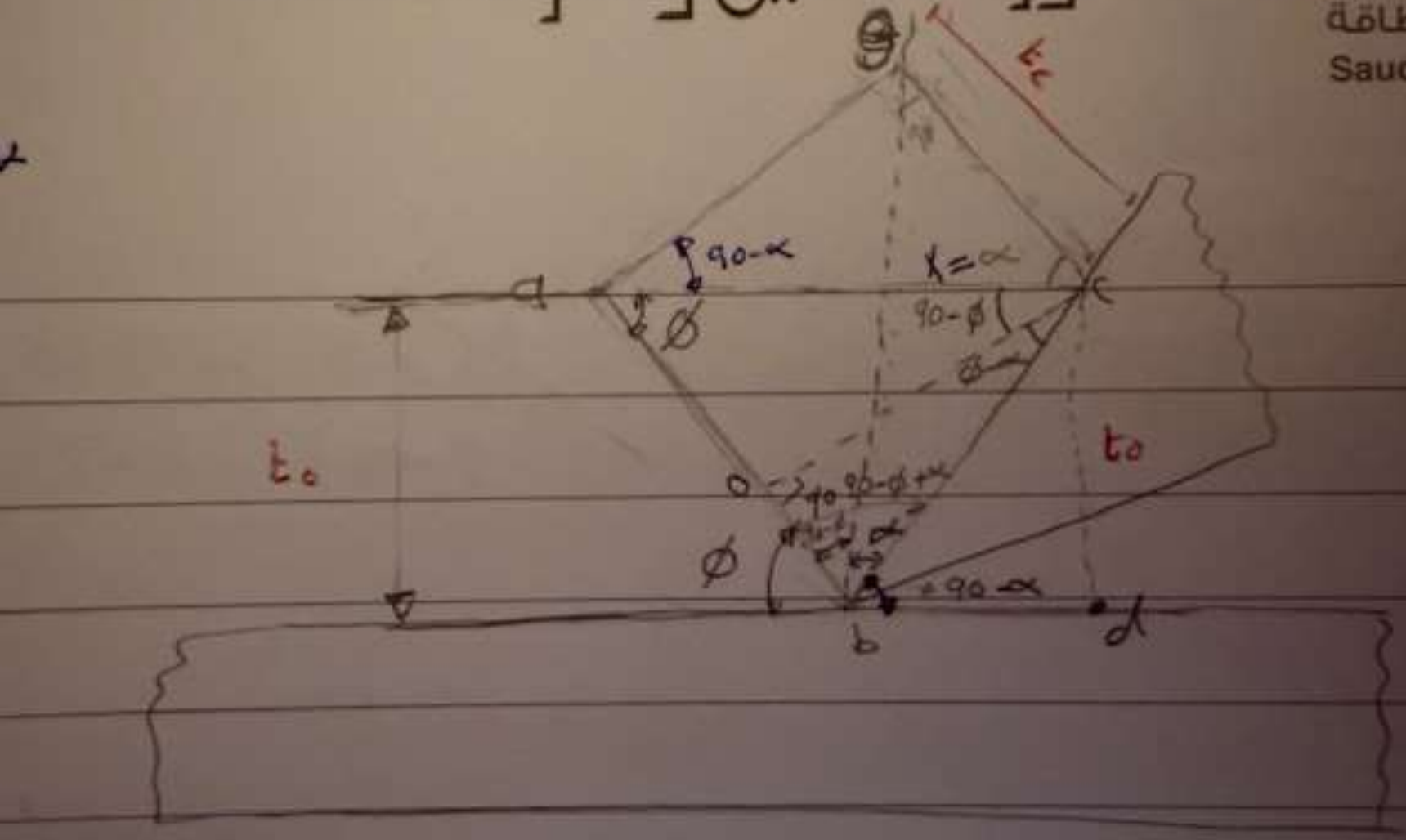
α = rake angle

Φ =Shear angle:

Cutting Ratio

دورة تدقيق وادارة الطاقة

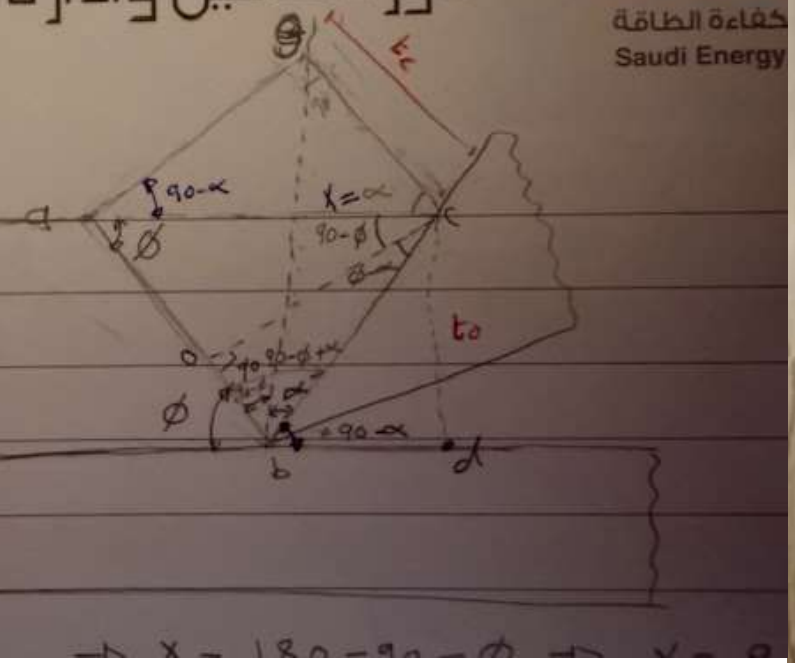
كفاءة الطاقة
Saudi Energy

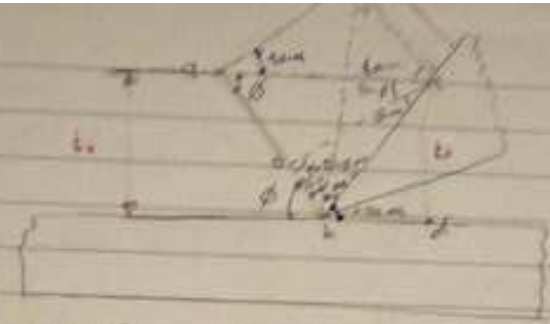


tting Ratio

دورة تدقيق وادارة

كفاءة الطاقة
Saudi Energy





For $\Delta aco \Rightarrow \phi + 90 + x = 180 \Rightarrow x = 180 - 90 - \phi \Rightarrow x = 90 - \phi$

For $\Delta boc \Rightarrow 90 + 90 - \phi + \alpha + x = 180 \Rightarrow x = \phi - \alpha$

- For $\Delta aco \Rightarrow \sin\phi = \frac{oc}{ac} \Rightarrow oc = ac \sin\phi$ ..... (1)

- For $\Delta ocb \Rightarrow \sin(90-\phi+\alpha) = \frac{oc}{bc} \Rightarrow oc = bc \sin(90-\phi+\alpha)$ .... (2)

- For $\Delta bcd \Rightarrow \sin(90-\alpha) = \frac{to}{bc} \Rightarrow bc = \frac{to}{\sin(90-\alpha)}$ ..... (3)

(2) و (3) عوض ⇒ substitute (3) in (2)

$oc = bc \sin(90-\phi+\alpha)$

$oc = \frac{to}{\sin(90-\alpha)} \sin(90-\phi+\alpha) \Rightarrow oc = \frac{to \sin(90-\phi+\alpha)}{\sin(90-\alpha)}$ ..... (4)

by equating (1) and (4)

$ac \sin\phi = \frac{to \sin(90-\phi+\alpha)}{\sin(90-\alpha)} \Rightarrow \frac{to}{ac} = \frac{\sin(90-\alpha) \sin\phi}{\sin(90-\phi+\alpha)}$

$to \sin(90-\phi+\alpha) = ac \sin\phi \sin(90-\alpha)$

$\sin(90-\phi+\alpha) = \sin(90-(\phi-\alpha)) = \sin(90-(\phi-\alpha)) = \cos(\phi-\alpha)$

$\sin(90-(\phi-\alpha)) = \cos(\phi-\alpha)$

ETAmax
Saudi Energy Efficiency

$$\therefore \frac{t_o}{ac} = \frac{\cos\alpha \sin\phi}{\cos(\phi - \alpha)} \quad \text{---- (5)}$$

From $\Delta agc \Rightarrow \cos\alpha = \frac{t_c}{ac} \Rightarrow ac = \frac{t_c}{\cos\alpha}$ ---- (6)

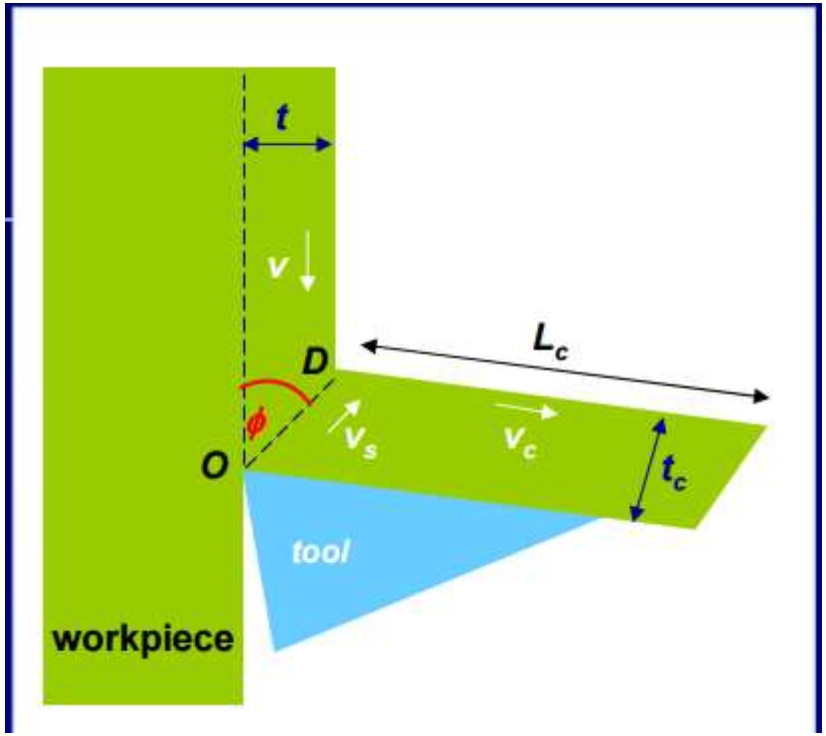
(5) في (6) عوض

$$\frac{t_o}{t_c/\cos\alpha} = \frac{\cos\alpha \sin\phi}{\cos(\phi - \alpha)}$$

$$\Rightarrow \frac{t_o \cos\alpha}{t_c} = \frac{\cos\alpha \sin\phi}{\cos(\phi - \alpha)}$$

$$\therefore \frac{t_o}{t_c} = \frac{\sin\phi}{\cos(\phi - \alpha)} = ٣$$

Calculation of the cutting ratio from chip length



Since ***volume*** is constant during plastic deformation, and chip width ***b*** is essentially constant,

$$L_w t b = L_c t_c b$$

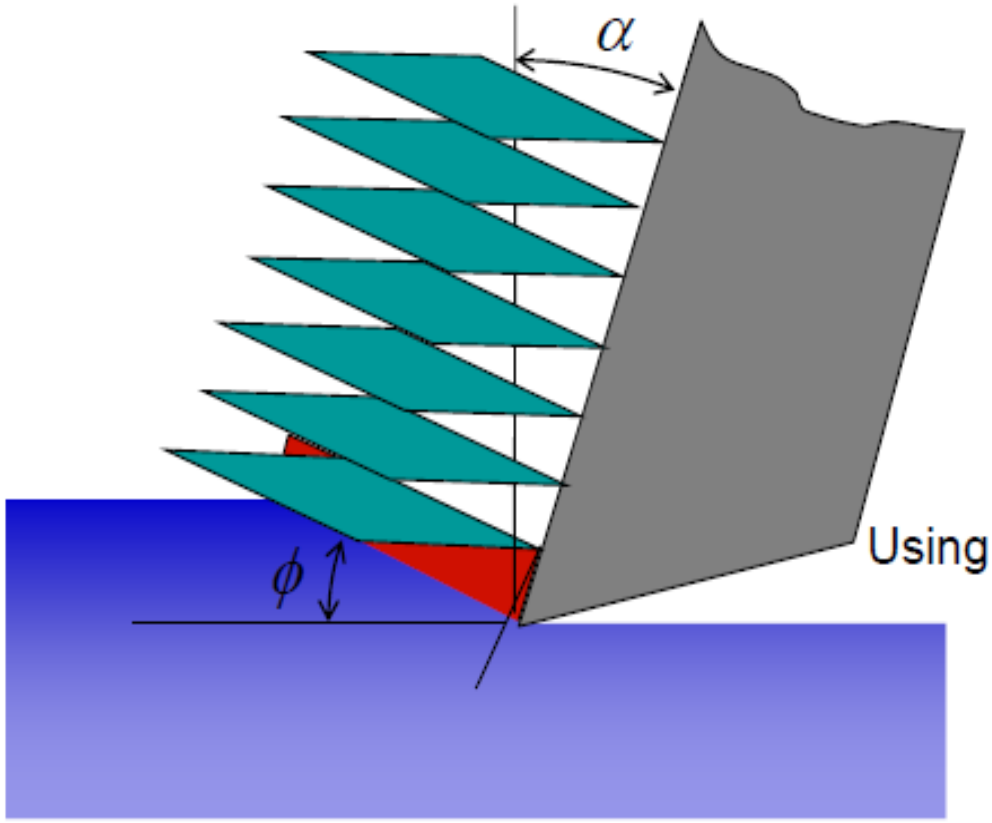
$$\frac{t}{t_c} = \frac{L_c}{L_w} = r$$

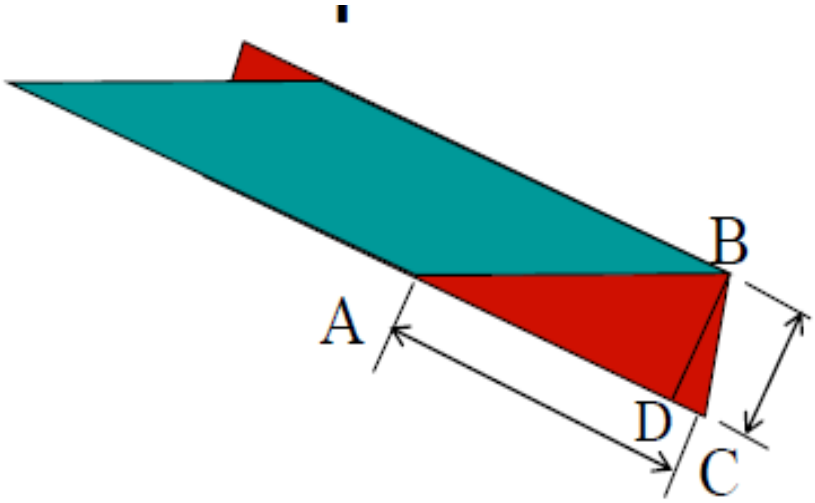
If t_c is unknown, it can be determined by measuring weight of the chip W_c and by knowing the density of the metal ρ

$$L_c t_c b = \frac{W_c}{\rho}$$

Shear Strain in chip

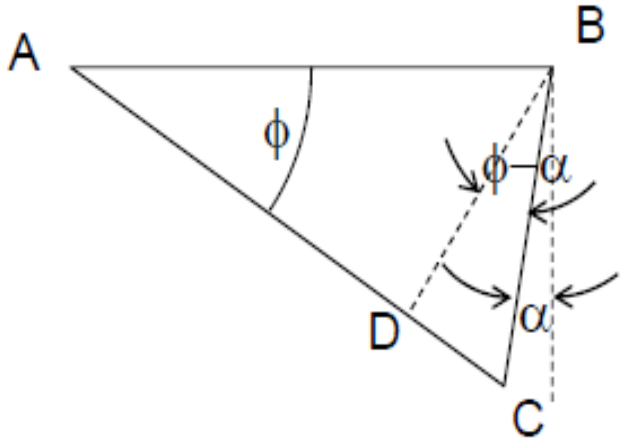
On the base of the figure below, the shear strain, that the material undergoes during cutting can be expressed as in the following equation:





Using $\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$

$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$



$$\gamma = \frac{AC}{BD} = \frac{DC + AD}{BD} = \tan(\phi - \alpha) + \cot\phi$$

$$= \frac{\sin(\phi - \alpha)}{\cos(\phi - \alpha)} + \frac{\cos\phi}{\sin\phi} = \frac{\cos\alpha}{\sin\phi\cos(\phi - \alpha)}$$

Shear Strain in chip

- Shear strain in machining can be computed from the following equation, based on the preceding parallel plate model:

$$\gamma = \tan(\phi - \alpha) + \cot \phi$$

- Where γ = shear strain, ϕ = shear plane angle, and α = rake angle of cutting tool.

- Note from this equation that high shear strains are associated with low shear angles and rake angles.

- Shear strain of 5 or higher have been observed in actual cutting operations. Thus, compared with forming and shaping processes, the material undergoes greater deformation as can be seen the next table

Shear Strain in chip

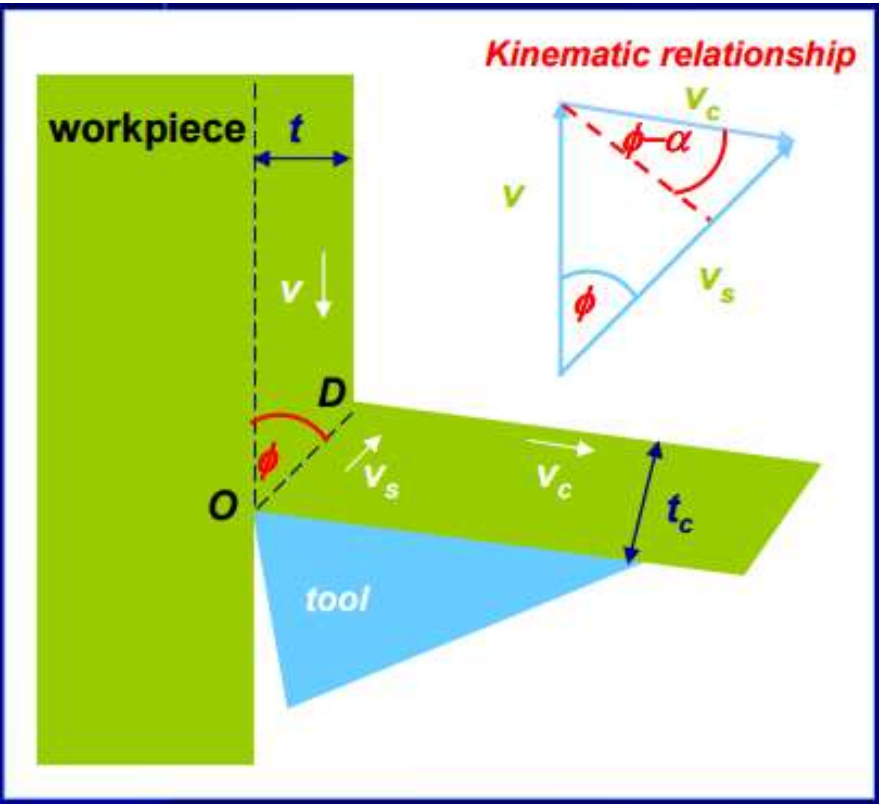
Typical Ranges of Strain, Deformation Speed, and Strain Rates in Metalworking Processes

Process	True strain	Deformation speed (m/s)	Strain rate (s^{-1})
Cold working			
Forging, rolling	0.1–0.5	0.1–100	$1–10^3$
Wire and tube drawing	0.05–0.5	0.1–100	$1–10^4$
Explosive forming	0.05–0.2	10–100	$10–10^5$
Hot working and warm working			
Forging, rolling	0.1–0.5	0.1–30	$1–10^3$
Extrusion	2–5	0.1–1	$10^{-1}–10^2$
Machining	1–10	0.1–100	$10^3–10^6$
Sheet-metal forming	0.1–0.5	0.05–2	$1–10^2$
Superplastic forming	0.2–3	$10^{-4}–10^{-2}$	$10^{-4}–10^{-2}$

The cutting speed

There are three velocities:

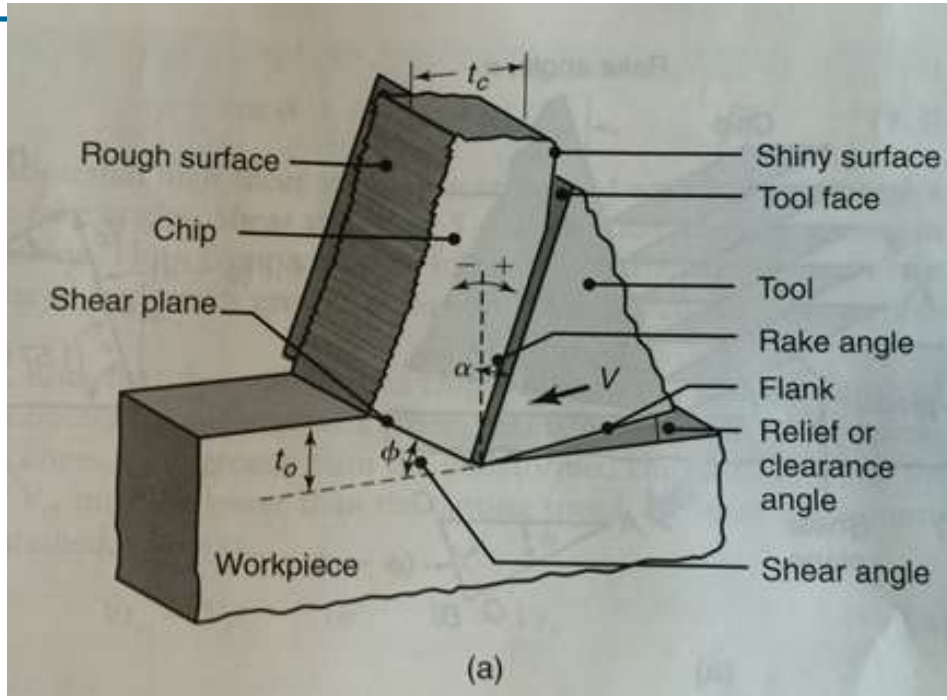
1) **Cutting speed** v, is the velocity of the tool relative to the workpiece.
2) **Chip velocity** v_c, is the velocity of the chip relative to the tool face
3) **Shear velocity** v_s, is the velocity of the chip relative to the work.



From ***kinematic relationship***, the vector sum of the cutting velocity and the chip velocity = the shear velocity vector.

Velocity

- From the figure, note that the undeformed chip thickness, t_o, and the depth of cut are the same parameter for the orthogonal cutting; this is not true for other operations.

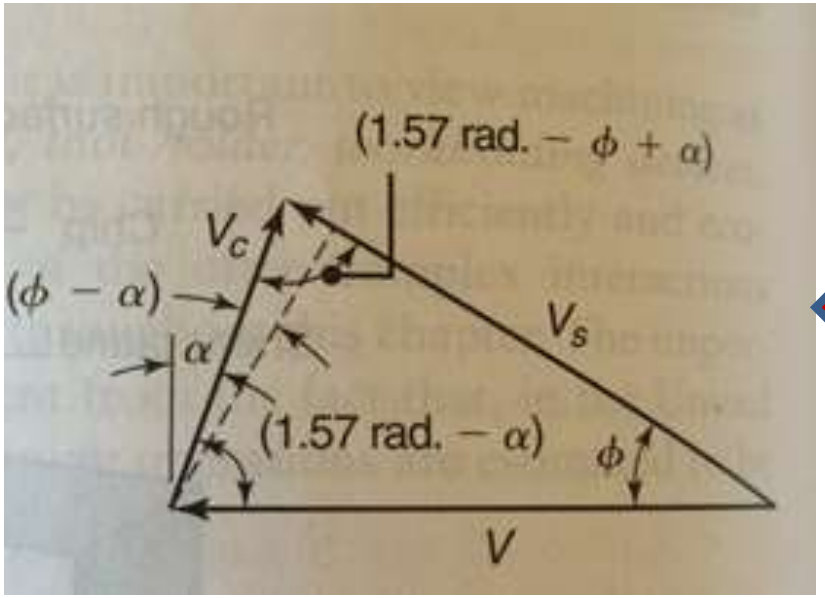


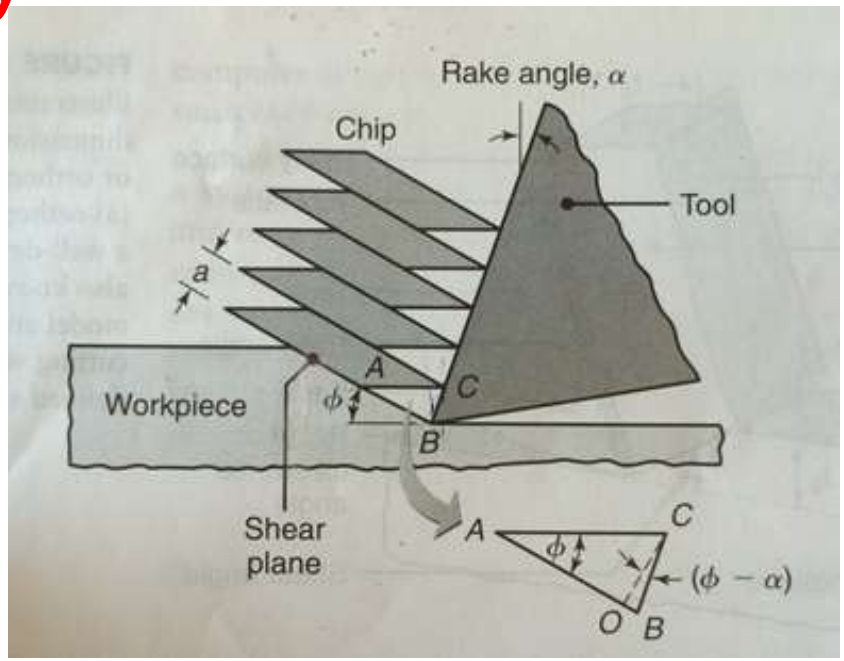
- Since the chip thickness, t_c, is greater than the undeformed chip thickness, t_o, the velocity of the chip, V_c, must be lower than the cutting speed, V. since mass continuity has to be determined, we have

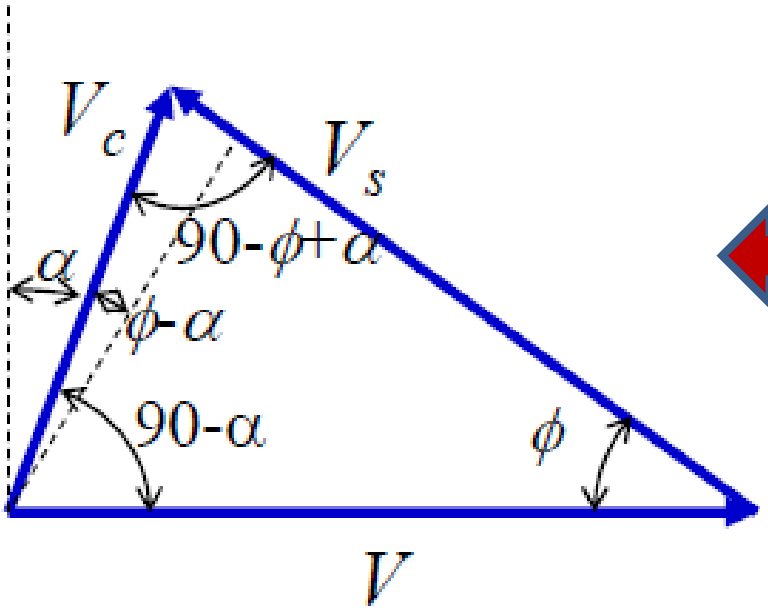
$$Vt_o = V_c t_c \quad \text{or} \quad V_c = Vr,$$

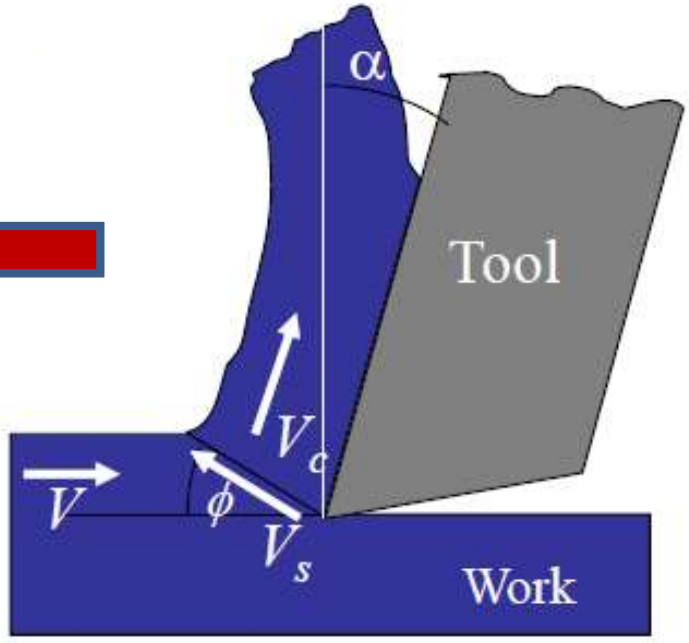
$$\Rightarrow \quad V_c = V \frac{\sin \phi}{\cos(\phi - \alpha)}$$

Velocity

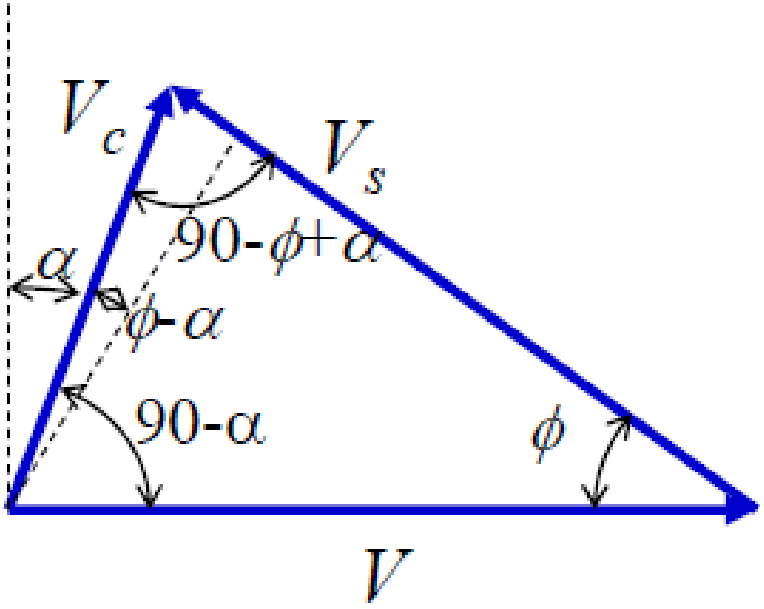

(1.57 rad. − ϕ + α)
V_c
(ϕ − α)
α
V_s
(1.57 rad. − α)
ϕ
V

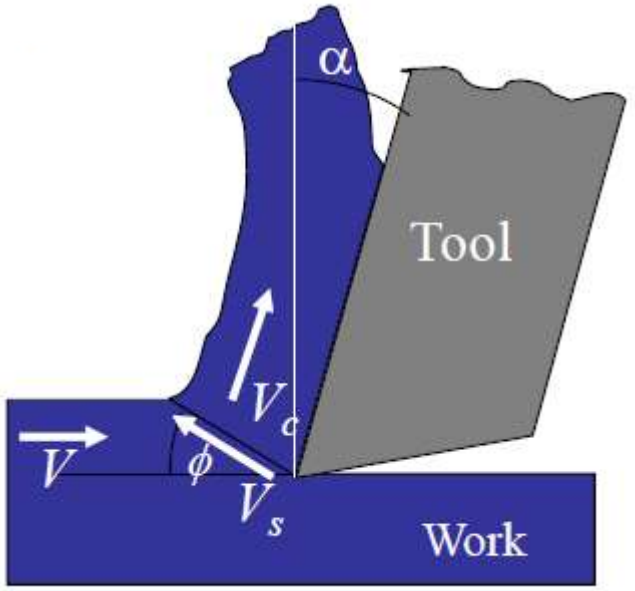

Rake angle, α
Chip
Tool
a
A
C
Workpiece
ϕ
B
Shear plane
A
C
ϕ
(ϕ − α)
O
B


V_c
V_s
α
90-ϕ+α
ϕ-α
90-α
ϕ
V


α
Tool
V_c
V
ϕ
V_s
Work

Velocity





$$\frac{V}{\sin\left(\frac{\pi}{2}-\phi+\alpha\right)}=\frac{V_s}{\sin\left(\frac{\pi}{2}-\alpha\right)}=\frac{V_c}{\sin\phi}$$

$$\frac{V}{\cos(\phi-\alpha)}=\frac{V_s}{\cos\alpha}=\frac{V_c}{\sin\phi}$$

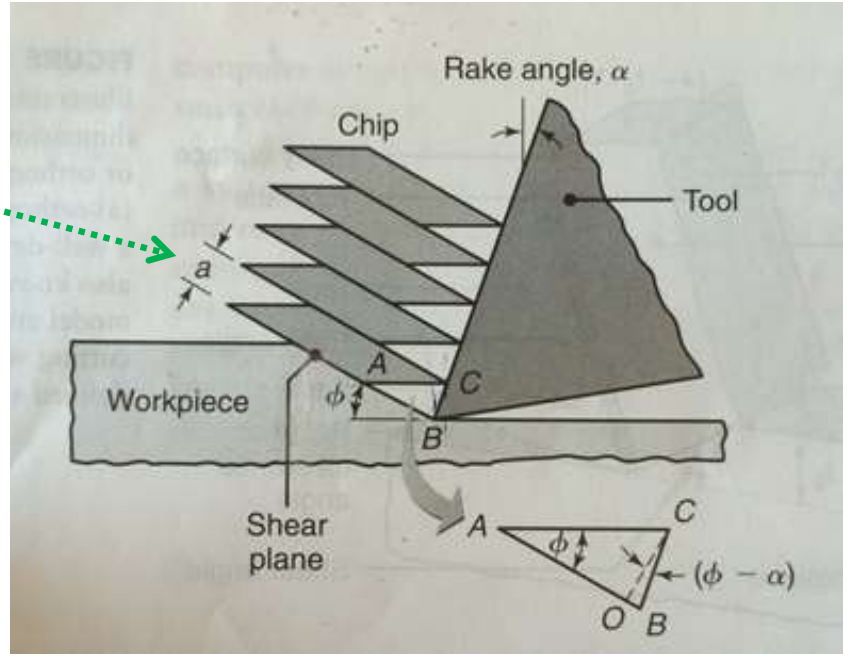
$$v_s=\frac{v\cos\alpha}{\cos(\phi-\alpha)}$$

Shear-strain rate

- The shear-stain rate is the ratio of V_s to thickness a of the sheared element (shear zone)

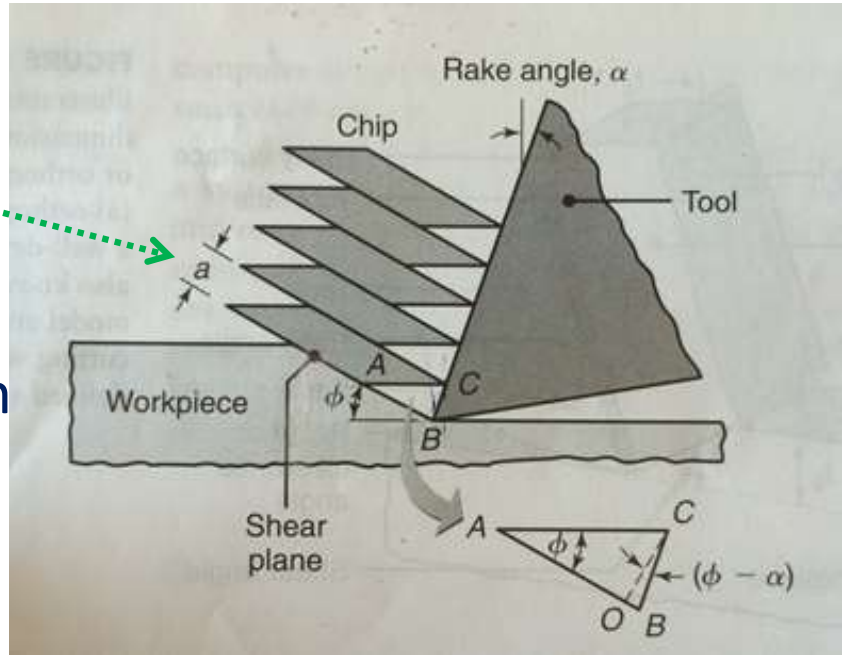
$$\dot{\gamma} = \frac{V_s}{a}$$

- The dimension [a] = OC
- where [a] is the estimate of the maximum value of the thickness of the shear zone



Shear-strain rate

$$\dot{\gamma} = \frac{V_s}{a}$$



- The magnitude of the shear-strain rate is very high. Why???????

1. Because of the very high deformation that occur at small area.
2. Because the magnitude of [a] is on the order of 10^{-2} to 10^{-3} mm. this range indicates that, even at low cutting speed (shearing speed), the shear-strain rate is very high, on the order of 10^3/s to 10^6/s

Shear Strain Rate in chip

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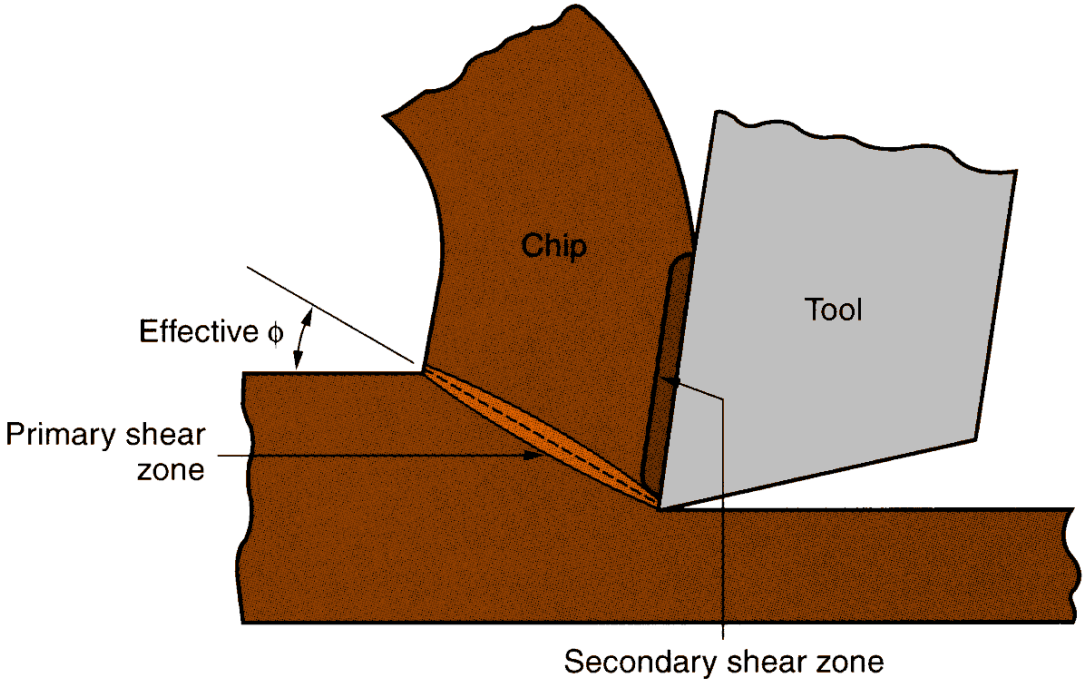
Machining	1–10	0.1–100	$10^3–10^6$
Sheet-metal forming	0.1–0.5	0.05–2	$1–10^2$
Superplastic forming	0.2–3	$10^{-4}–10^{-2}$	$10^{-4}–10^{-2}$

Chip morphology

- The type of chips produced (chip morphology) significantly influences ***surface finish*** and ***integrity of the machined part***.
- The chip has two faces:

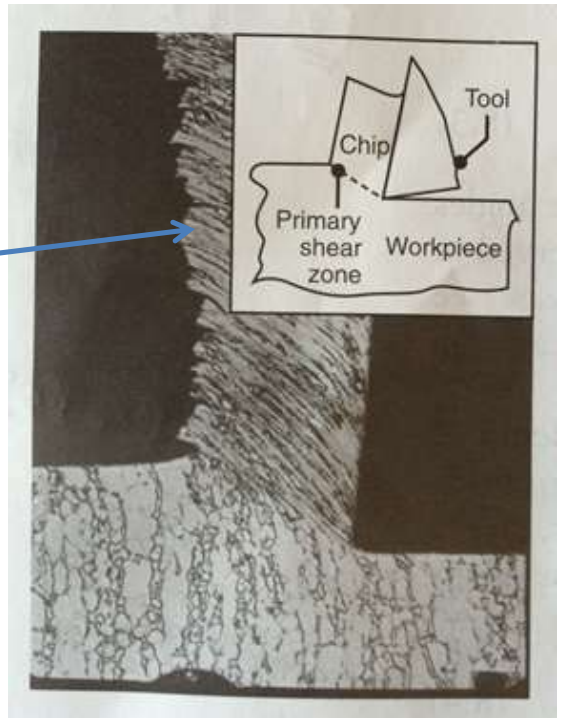
1. One surface has been in contact with the rake face of the tool
2. The other surface is from the original surface of the workpiece.



Chip morphology

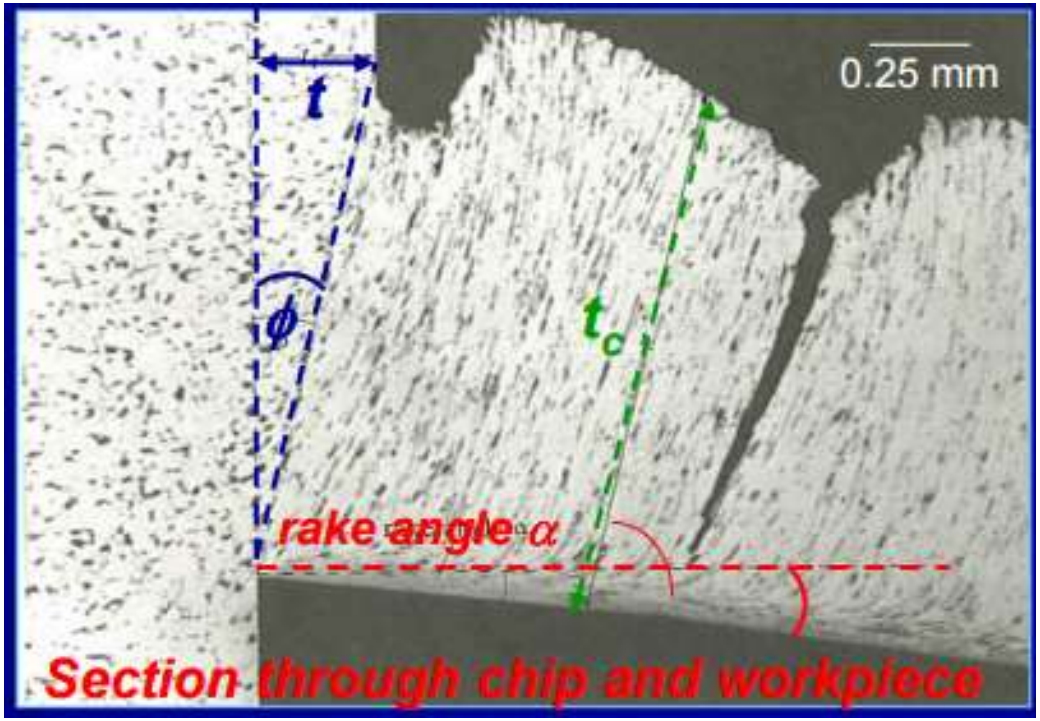
- The tool side of the chip surface is ***shiny*** or ***burnished***, caused ***by the rubbing of the chip as it climbs up to the tool surface.***

- The other surface of the chip has not come into contact with any solid body. So it has ***a jagged***, ***steplike*** appearance, ***which is due to the shearing mechanism of chip formation.***



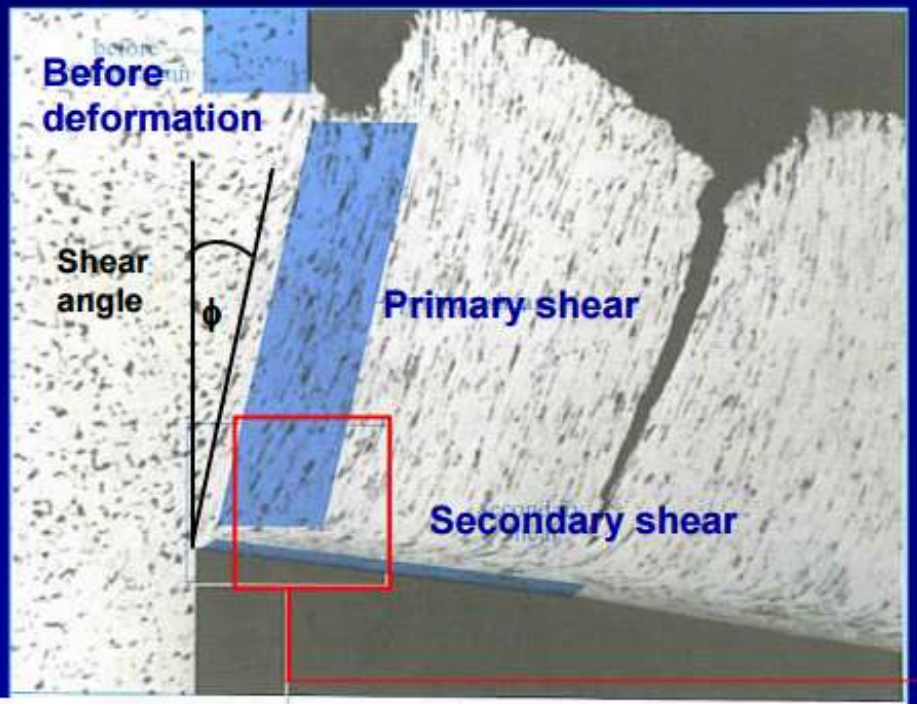
Chip formation

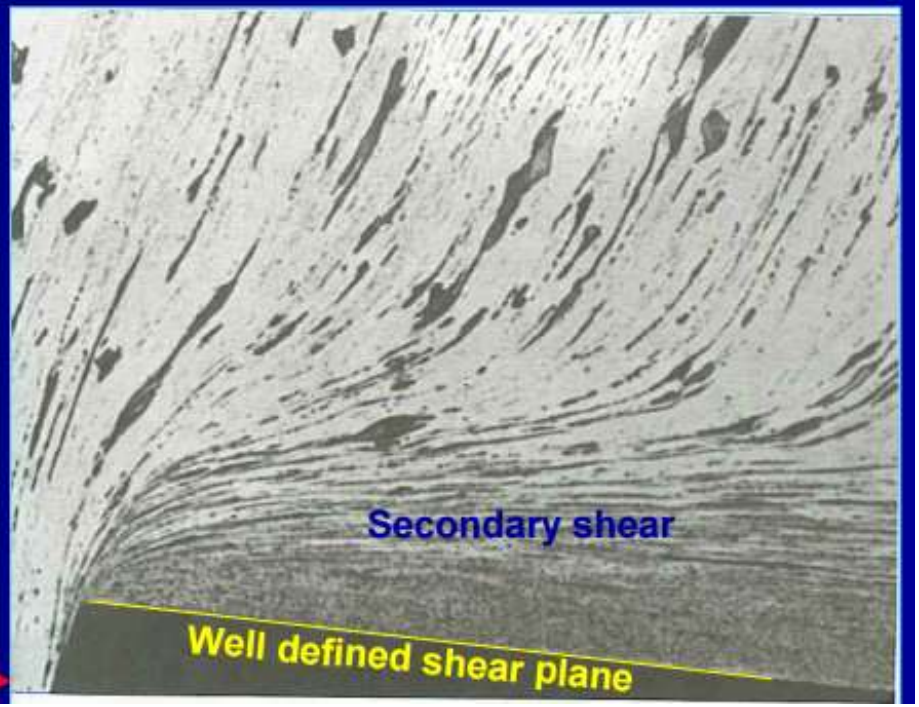
- The tool removes material near the surface of the workpiece by ***shearing it to form the chip***.
- Material with thickness ***t*** is sheared and travels as a ***chip*** of thickness t_c along the ***rake face*** of the tool.



Two basic deformation zone:

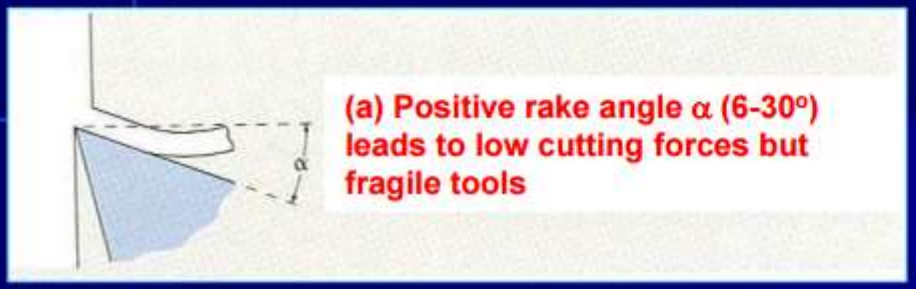
- The entire chip is deformed as it meets the tool, known as ***primary shear***. Shear plane angle is ϕ.
- Localised region of intense shear occurring due to the friction at the rake face, known as ***secondary shear***.





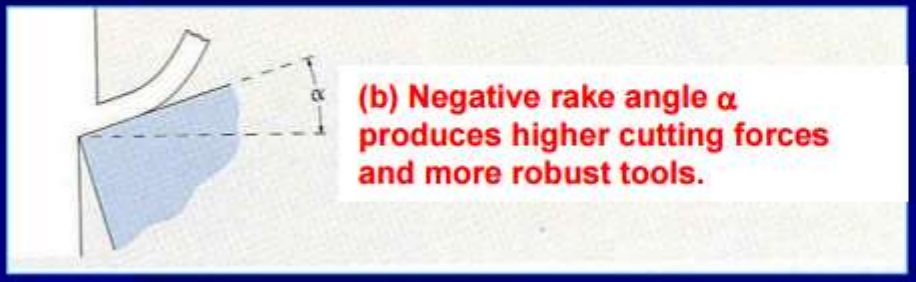
Chip formation

- The amount of ***primary shear*** is related to the ***rake angle*** α.



(a) Positive rake angle α (6-30°) leads to low cutting forces but fragile tools

(a) If α is a large ***positive*** value, the material is deformed less in the chip.



(b) Negative rake angle α produces higher cutting forces and more robust tools.

(b) If α is a ***negative*** value, the material is forced back on itself, thus requiring ***higher cutting forces***.

The relationship between ***rake angle***, ***shear angle***, and ***chip thickness ratio***, *r* can be derived as follows

$$\tan\phi = \frac{r\cos\alpha}{1 - r\sin\alpha}$$

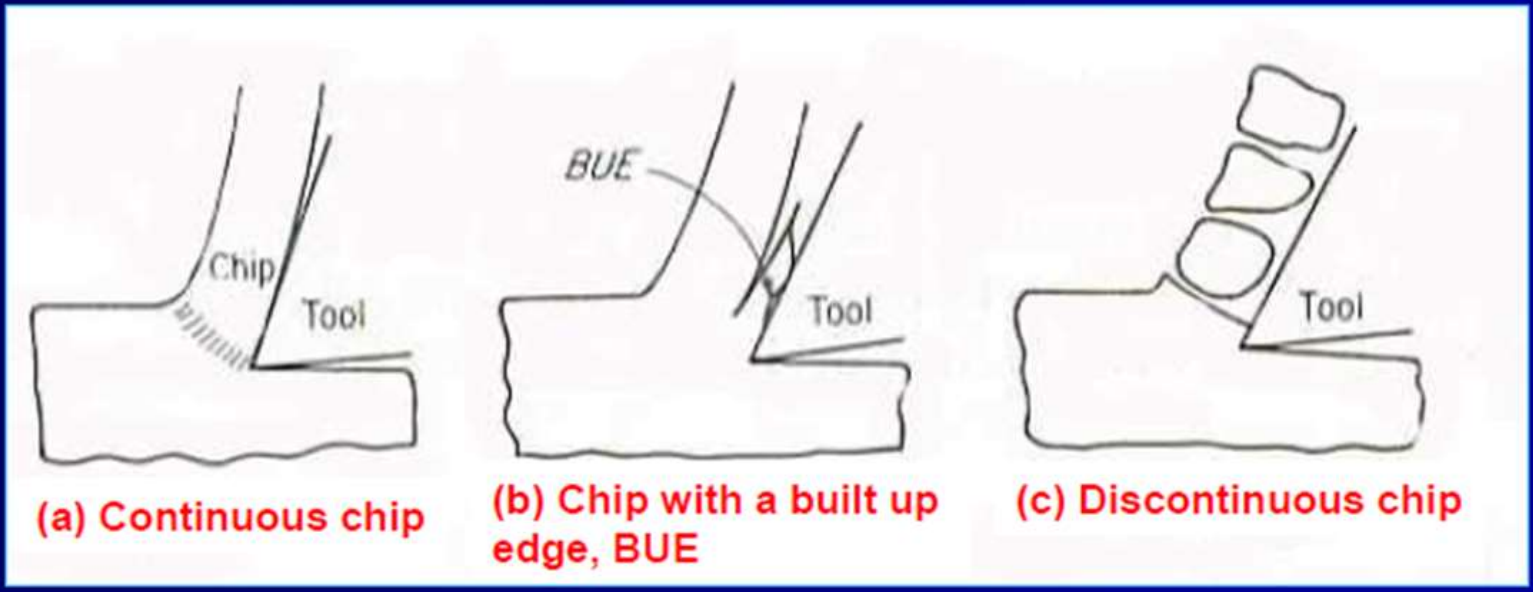
So the shear angle is controlled by rake angle

Four Basic Types of Chip in Machining

1. Discontinuous chip
2. Continuous chip
3. Continuous chip with Built-up Edge (BUE)
4. Serrated chip

Type of machining chips

Three general classifications of chips are formed in the machining process.

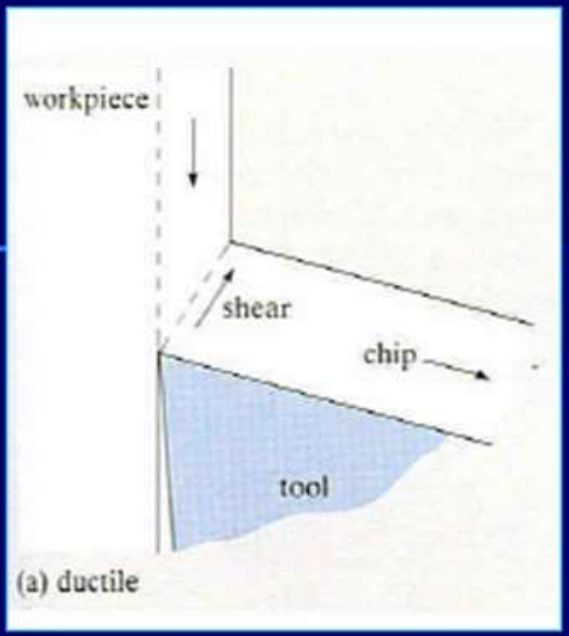


(a) Continuous chip

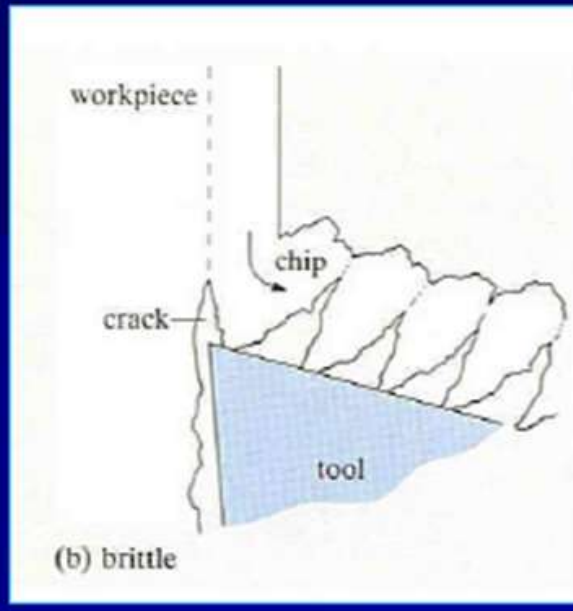
(b) Chip with a built up edge, BUE

(c) Discontinuous chip

Continuous chips



Discontinuous chips



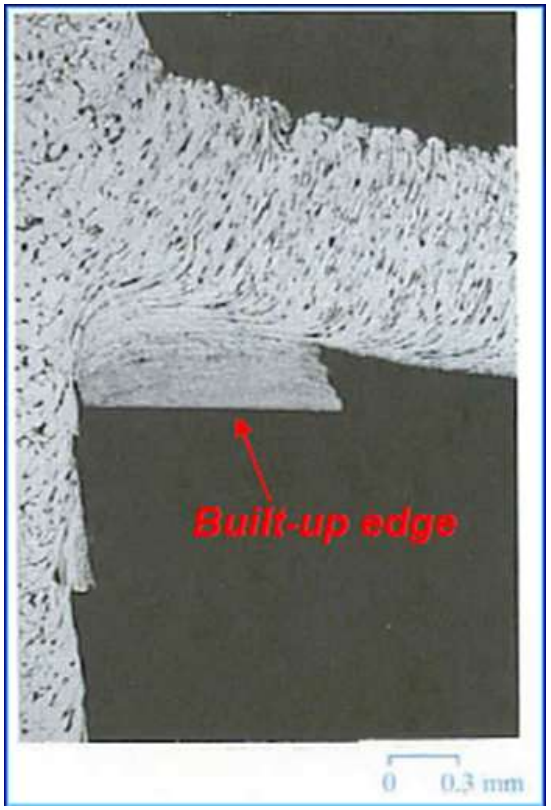
Continuous chip is characteristic of cutting ***ductile materials*** under steady stage conditions.
However, long continuous chips present handling and removal problems in practical operation.

- required chipbreaker.

Discontinuous chip is formed in ***brittle materials*** which cannot withstand the high shear strains imposed in the machining

Chip with a built-up edge (BUE)

- The formation of BUE is due ***to work hardening*** in the secondary shear zone at low speed
- The accumulation of the chip material is known as ***a built-up edge*** (BUE).
- Under conditions where the ***friction*** between the chip and the rake face of the tool is high, the chip may weld to the tool face.
- The BUE act as a substitute cutting edge.

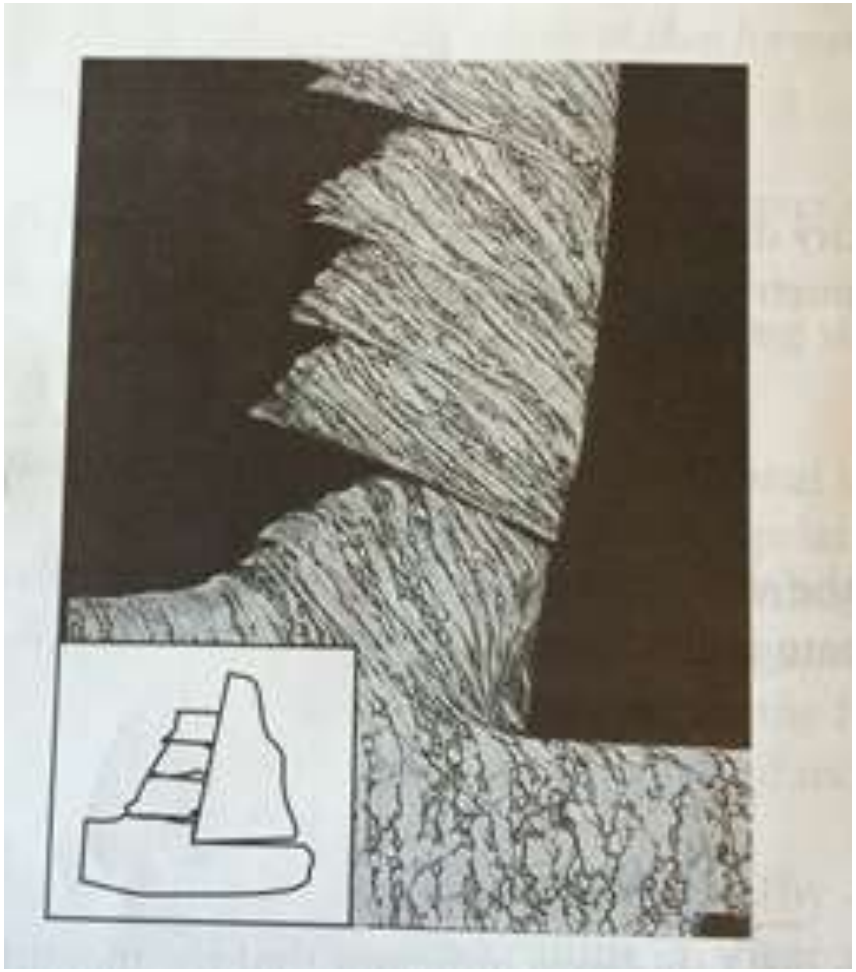


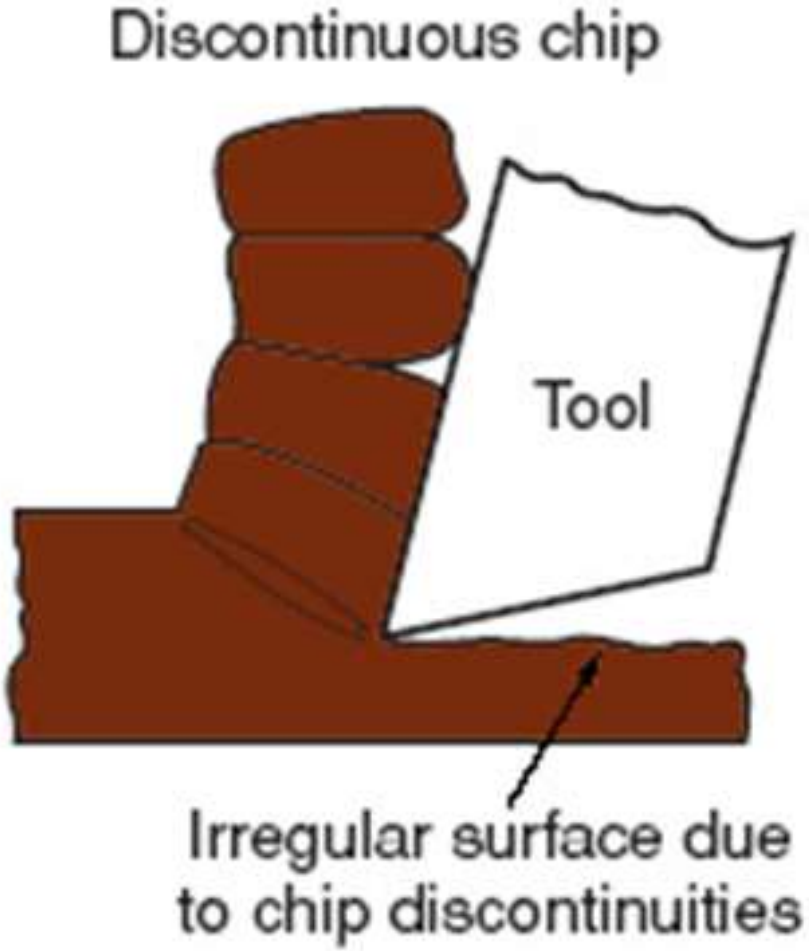
Chip formation with a built-up edge.

Poor texture on the surface

Discontinuous Chip

- Discontinues chips consist of segments that may be either firmly or loosely attached to each other.





Discontinuous Chip

- These chips usually form under the conditions:

1. The workpiece material is *brittle*, [because brittle material can not undergo the high shear strains involved in cutting].
2. The workpiece material *contains hard inclusions* and *impurities.* ***Impurities and hard particles act as sites for cracks, thereby, producing discontinuous chips***. As expected, a large depth of cut increases the probability of such Chip
3. The depth of cut (undeformed chip thickness) is larger than rake angle.
4. There is a lake of an effective cutting fluid.
5. The cutting speed is very low
6. The machine tool has low stiffness and poor damping ***of the cutting tool.***

Discontinuous Chip

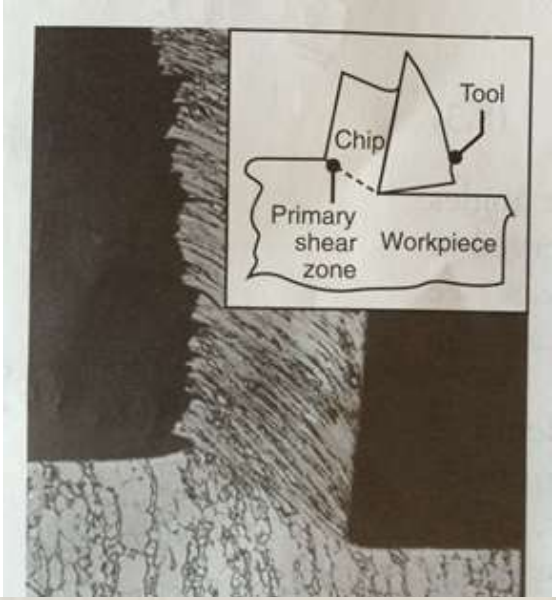
- Because of the discontinuous nature of chip formation, forces continually vary during cutting. Consequently, *the stiffness of cutting tool holder*, *the workpiece holding devices*, and *the machine tool* is important in cutting with discontinuous chips as well as serrated-chip formation. If not sufficiently stiff, the machine tool may begin to vibrate and chatter. This effect, ***in turn, adversely affects the surface finish and dimensional accuracy of the machine part*** and ***may even cause damage or excessive wear of the cutting tool, as well as the machine tool itself.***

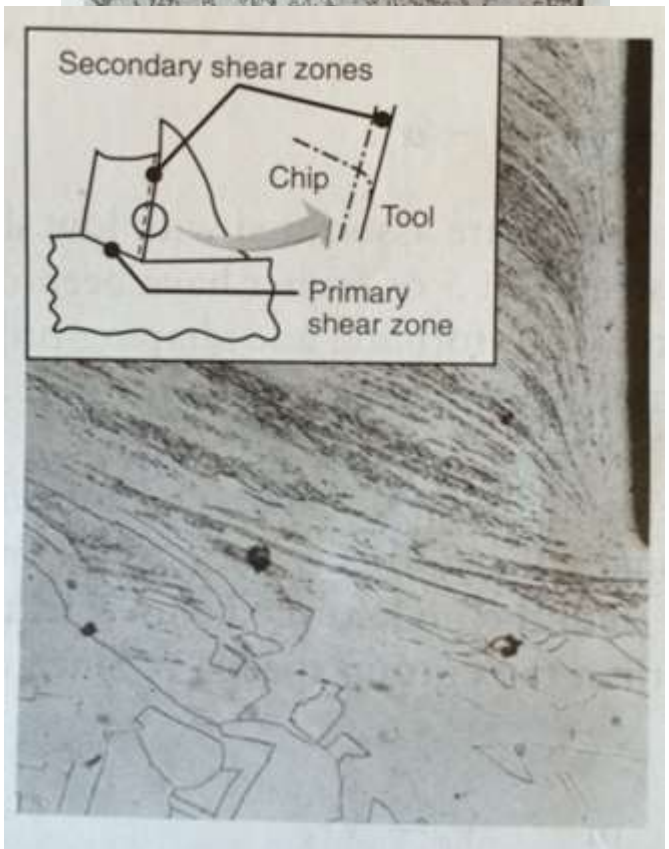
Continuous Chip

- Continuous chips are typically formed in ***Ductile work materials*** (e.g., low carbon steel) at:

1. High cutting speed and/or high rake angles
2. Small feeds and depths
3. Sharp cutting edge on the tool
4. Low tool-chip friction

- The deformation of the material takes place along a very narrow zone, called primary shear zone. Such chips also may develop a secondary shear zone at tool-chip interface, caused by friction. The secondary zone becomes thicker as tool-chip friction increases





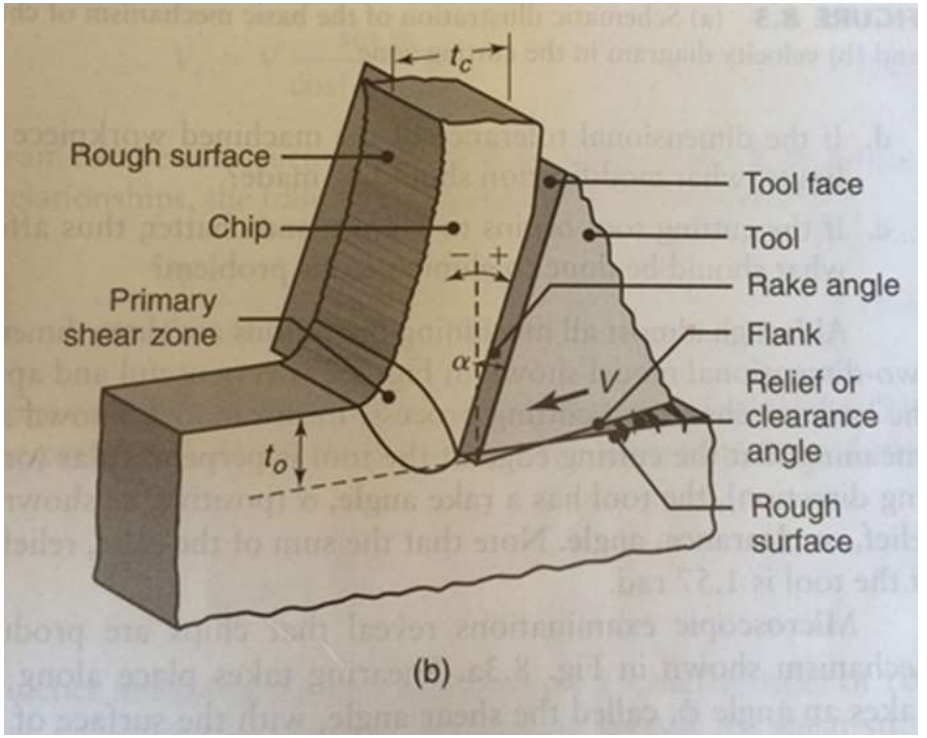
Continuous Chip

- CONTINOUS chips generally produce *a good surface finish*, but they are not always *desirable*. Why ?????
  - *Particularly, in computer-controlled machined tools, because they tend to get tangled around the tool, and the operation has to stopped to clear away the chips.*
  - This problem can easily be solved with chip breakers features on cutting tool.
- Because of the shear strain that the chip is subjected to. The chip will be in a state of strain hardening. As a result of that the chip becomes harder and stronger, and less ductile, than the original workpiece material.
- The increase in the hardness and strength of the chip depends on the magnitude of the shear strain. As the rake angle decreases, the shear strain increases, and thus the chip becomes stronger and harder

$$\gamma = \frac{AC}{BD} = \frac{DC + AD}{BD} = \tan(\phi - \alpha) + \cot\phi$$

Continuous Chip

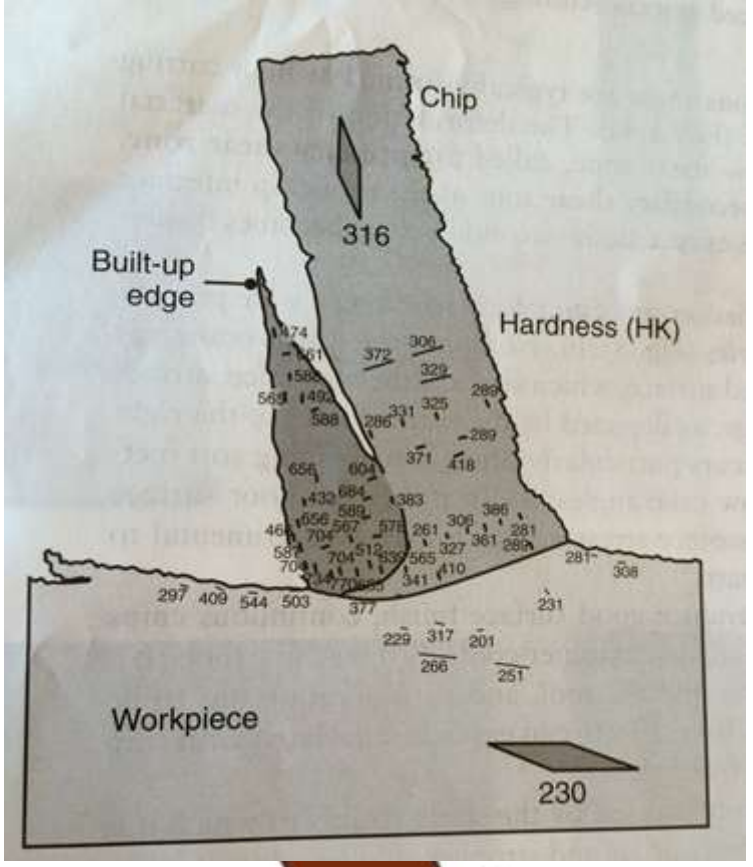
- In continuous chips, deformation may also take place along a wide primary shear zone with curved boundaries (figure b).
- Note that the lower boundary of the zone is below the machined surface, which subjects the machined surface to distortion and possible damage

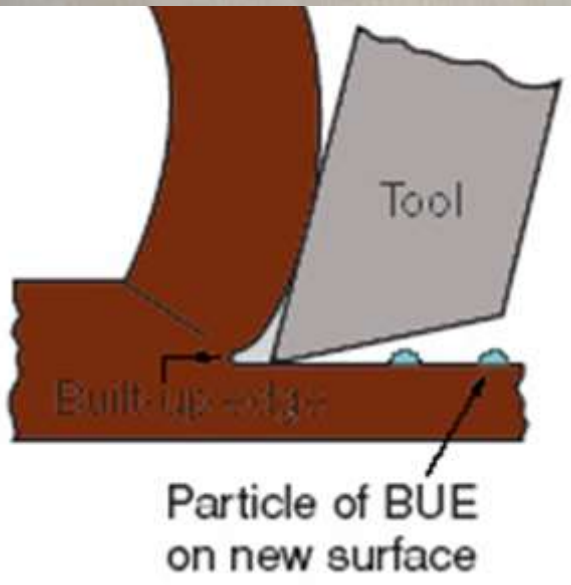


- This situation occurs particularly often in machining soft metals at low cutting speeds and low rake angles; it can produce a poor surface finish and also induce residual surface stresses, which may be detrimental to the properties of the machined part

Continuous with BUE

- A built-up edge (BUE) chip may form with *a ductile material* at *the tip of the cutting tool.*
- It consists ***of layers of material from the work piece that are gradually deposited on the tool*** (***hence the term built-up***).
- ***As it becomes larger***, the BUE becomes unstable and eventually breaks up.
- The upper portion of the BUE is carried a way on the tool side of the chip, and the lower portion is deposited randomly on the machine surface.
- The process of BUE formation and breakup is repeated continuously during the cutting operation.

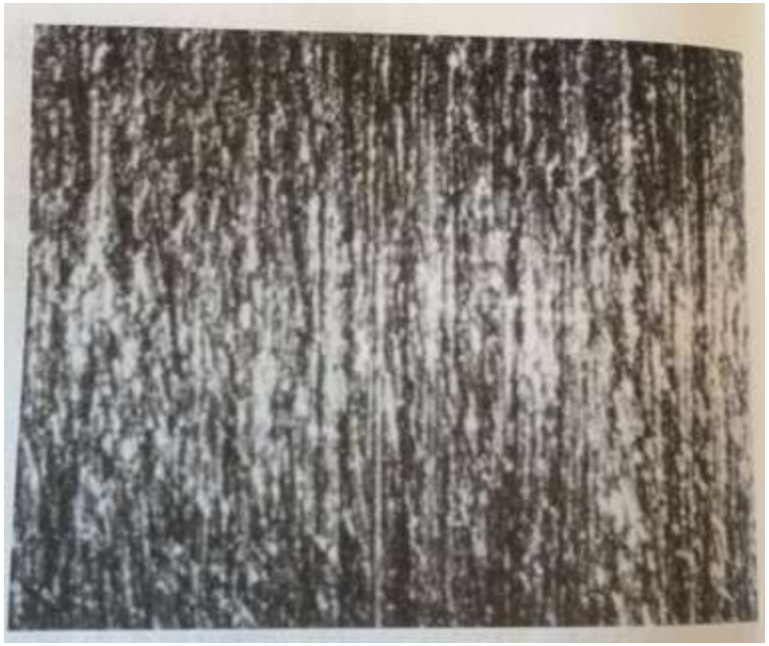




Continuous with BUE

- The built-up edge is one of the important factors *that adversely affects surface finish* and *integrity of the machined part*. In which the BUE changes the geometry of cutting operation. How????
- Large tip radius of the BUE will produce a rough surface finish, ***because of work hardening and deposition of successive layers of material, BUE hardness increase significantly.***
- Although BUE is generally undesirable, but a thin stable BUE (dose not break) is generally regarded as desirable because it protect the tool surface

Surface finish in face milling



Surface finish in turning steel with built-up edge

Continuous with BUE

- Although the exact mechanism of BUE formation is not clearly understood, investigations have identified the factors that contribute to the formation of BUE:

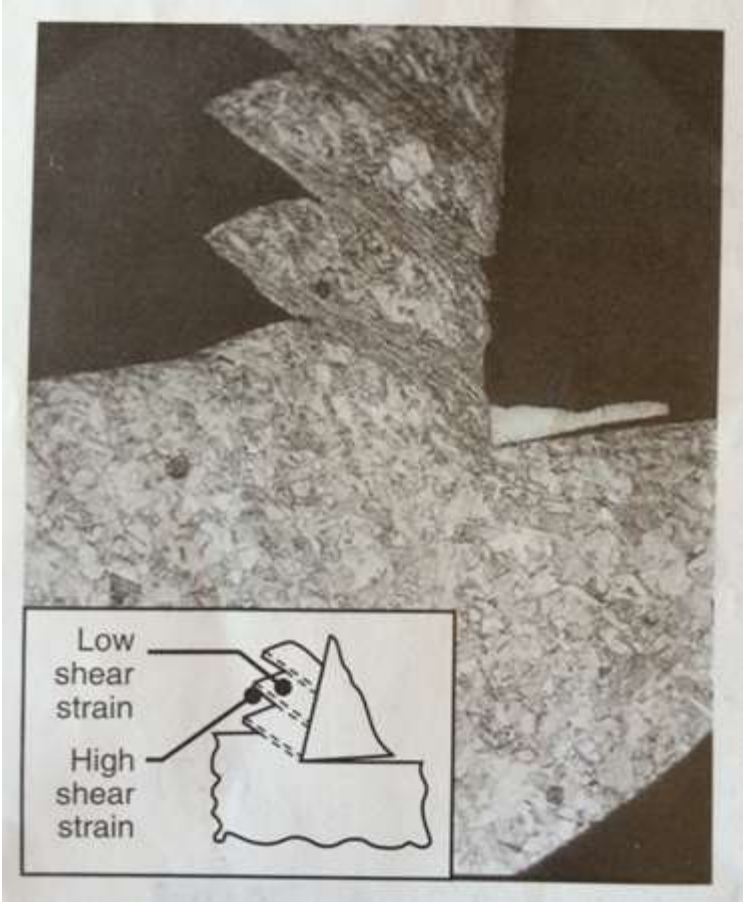
1. Adhesion of the workpiece material to rake face of the tool, the strength of this bond depending on the affinity of the workpiece and tool materials [ceramic cutting tools, for example, have much lower affinity to form BUEs than do tool steels].
2. Growth of the successive layers of adhered metal on the tool.
3. Tendency of workpiece material for strain hardening; the higher the strain hardening exponent, n, the higher is the probability for BUE formation. [the separation of more hardness material from the less one]

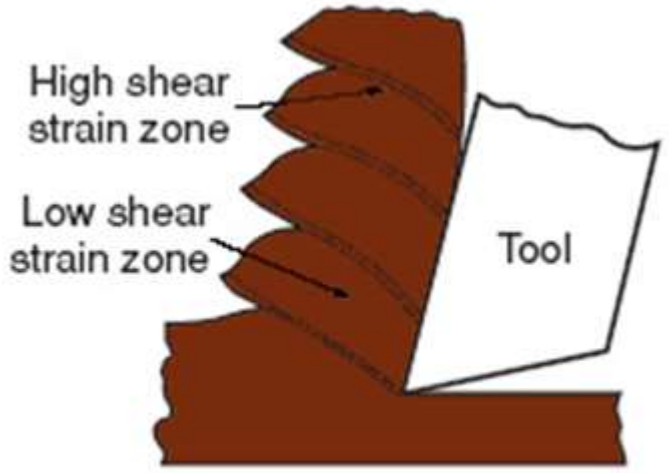
- The Buit-up edge decreases or is eliminated as :

1. The cutting speed, v, increases
2. The depth of cut, t_o,decreases
3. The rake angle, α, increases
4. Tip radius of the tool decreases

Serrated Chip

- Serrated chip, also called ***segmented*** or ***Semicontinuous*** or ***nonhomogeneous*** chips.
- They are Semicontinuous chips with zones of ***low shear*** and ***high shear strain.***
- The chips have the appearance of saw teeth (***hence the term serrated***)
- Metals, such as titanium, with low thermal conductivity and strength decreases sharply with temperature exhibit this behavior.



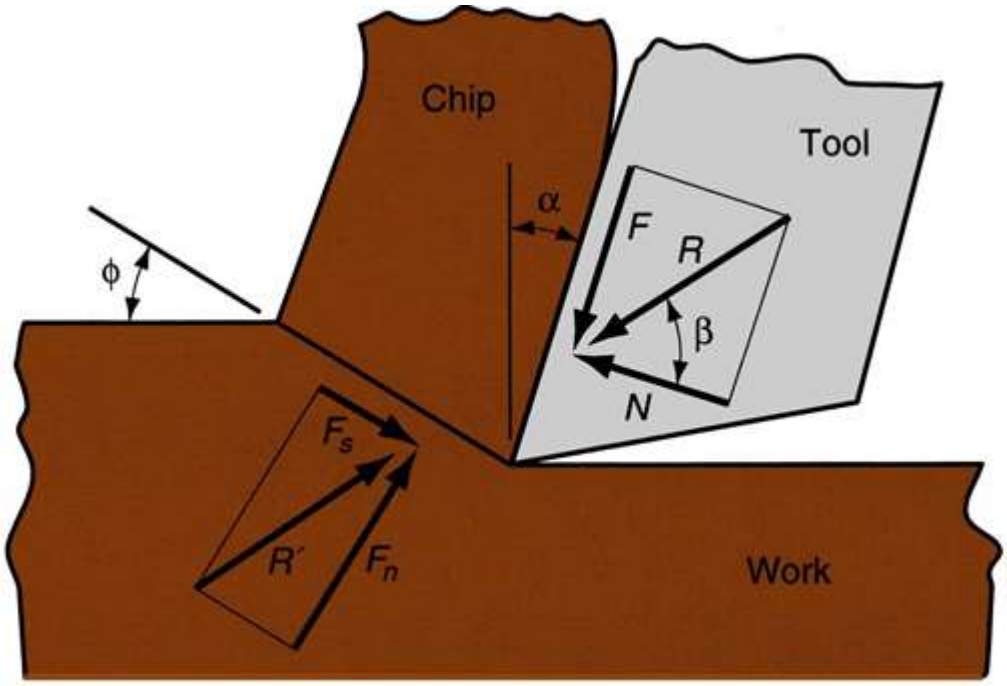


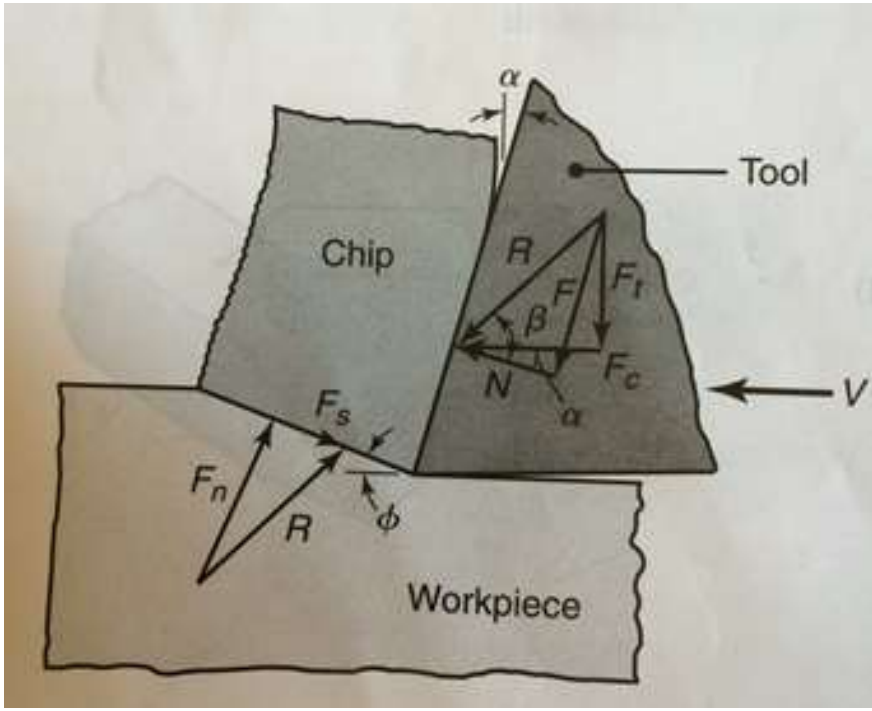
Forces Acting on Chip (all the forces)

- Analysis

– Forces in cutting:

- Forces acting on the chip:





❑ Two types of forces

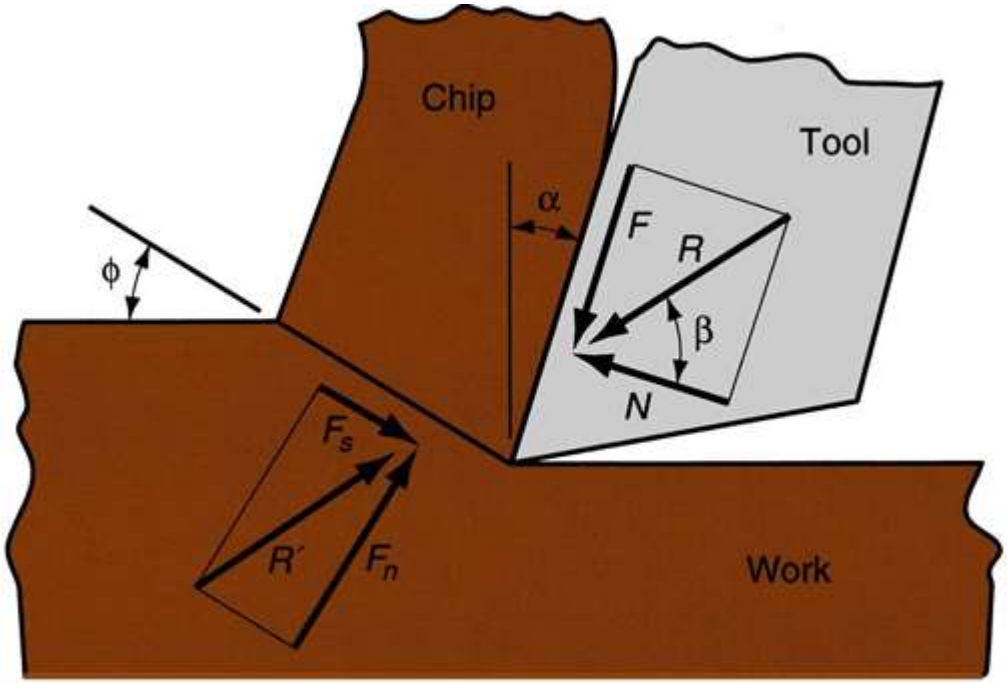
1. Forces can not be measured
2. Forces can be measured using dynometer

Forces Acting on Chip

- Analysis

– Forces in cutting:

- Forces acting on the chip:



F : Friction force on tool chip interface (friction coefficient is μ)

N : Normal force to friction

F_s : Shear force

F_n : Normal force to shear

R : Resultant of the forces F and N

R' : Resultant of the forces F_s and F_n

- F, N, F_s, and F_n cannot be directly measured

- ❑ The friction force, ***F***, along the tool-chip interface
- ❑ The normal force, ***N***, which is perpendicular to the interface
- ❑ These two forces produce the resultant force, R.
- ➢ The shear force, ***Fs***, along the shear plane
- ➢ Normal force to shear, ***F_n***, which is perpendicular to shear plane
- ➢ These two forcese produce the resultant force, ***R'***.

Forces Acting on Chip

FORCES INCLUDED IN METAL CUTTING

- F_s , Resistance to shear of the metal in forming the chip. It acts along the shear plane.
- F_n , 'Backing up' force on the chip provided by the workpiece. Acts normal to the shear plane.
- N, It at the tool chip interface normal to the cutting face of the tool and is provided by the tool.
- F, It is the frictional resistance of the tool acting on the chip. It acts downward against the motion of the chip as it glides upwards along the tool face.

Resultant Forces

- Vector addition of F and N = resultant R
- Vector addition of F_s and F_n = resultant R'
- Forces acting on the chip must be in balance:
  - R' must be equal in magnitude to R
  - R’ must be opposite in direction to R
  - R’ must be collinear with R

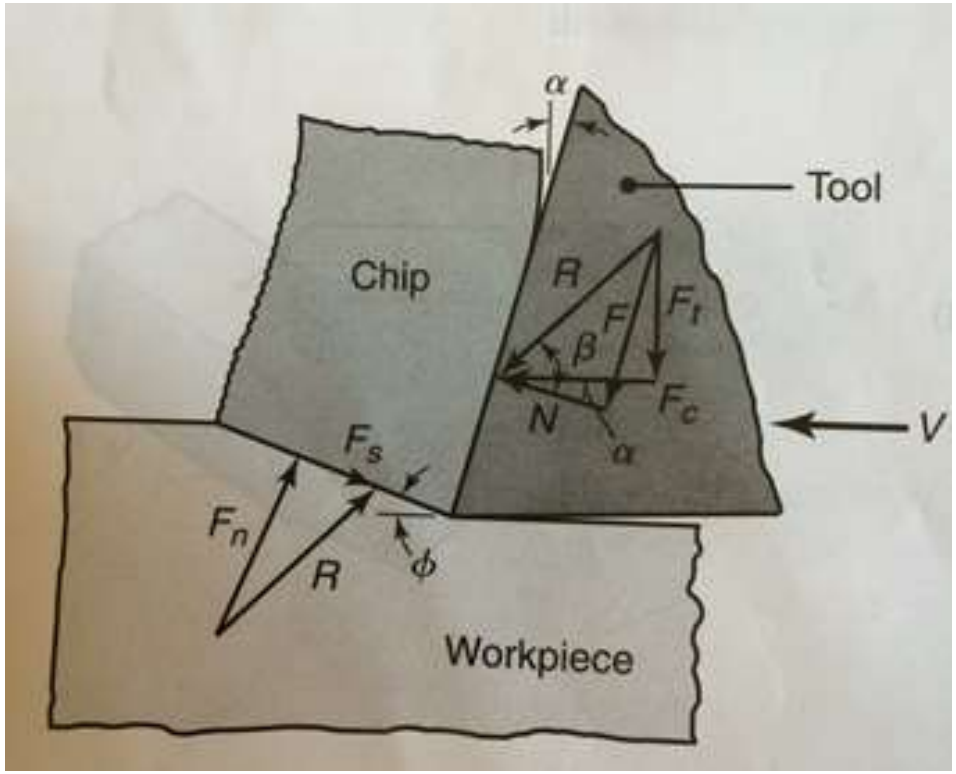
Forces Acting on Chip

- Analysis

– Forces in cutting:

- Forces acting on the chip:

- *The cutting force,* ***Fc****,* acts in the direction of the cutting speed, ***v****,* and supply the energy required for the machining operation.
- *The thrust force (feed force),* ***Ft****,* acts in the direction normal to cutting velocity, that is perpendicular to the workpiece.
- These two forces produce the resultant force, ***R”*** .



- Forces acting on the tool that can be measured:
  - Cutting force F_c and Thrust force F_t

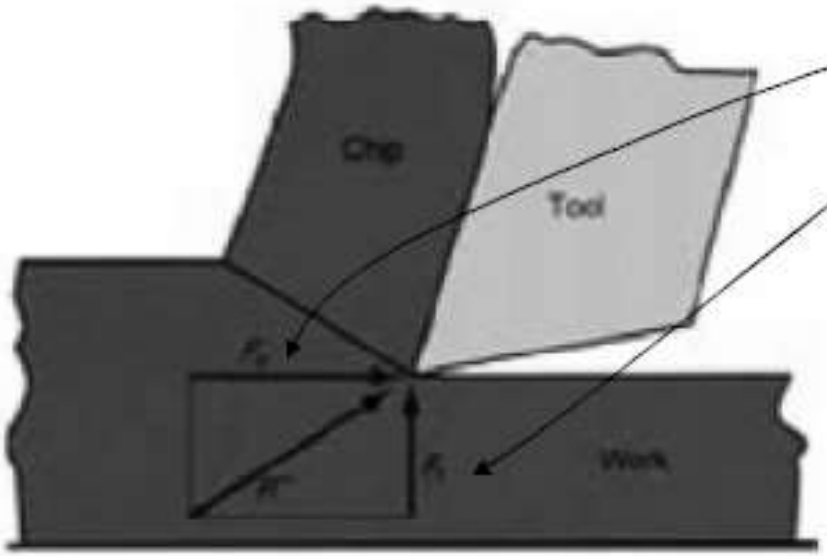
Need of analysis of forces

- Analysis of cutting forces is helpful as :

1. Determination of power consumption [estimation of cutting power consumption, which enables selection of the power source(s)(motor selection) during design of machine tool].
2. Knowing cutting forces is helpful in designing the machine in regard the stiffness and damping
3. To obtain maximum productivity[if you know forces acting on cutting tool we can easily predict life of the tool so we can obtain balance between life of the tool and rate of cutting in order to obtain maximum productivity].

Forces Acting on Chip

- Analysis
  - Forces in cutting:
    - Forces acting on the tool:



F_c : cutting force

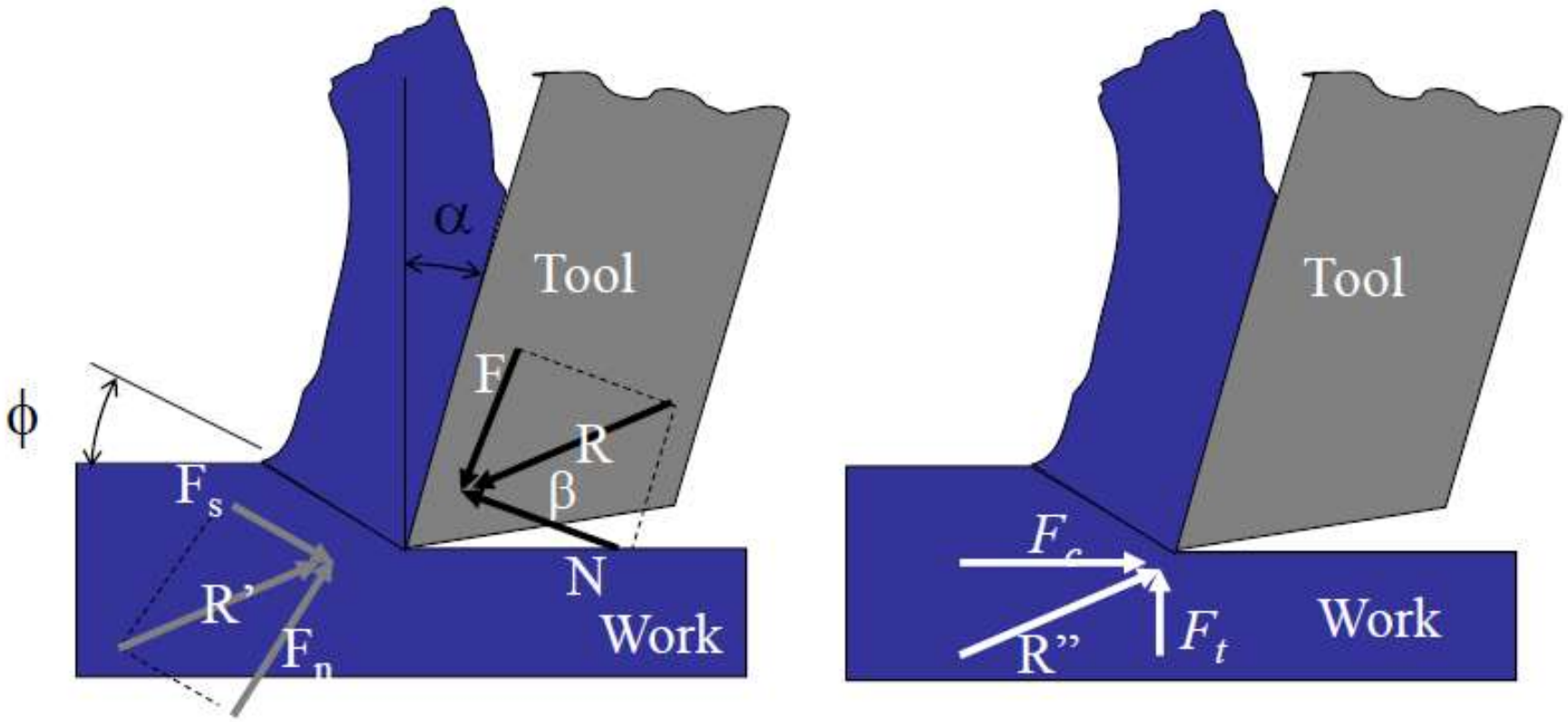
F_t : thrust force

R'': Resultant of F_c and F_t

These forces can be measured directly!

Note that R, R′, and R″ must be equal in magnitude

Force Relationships

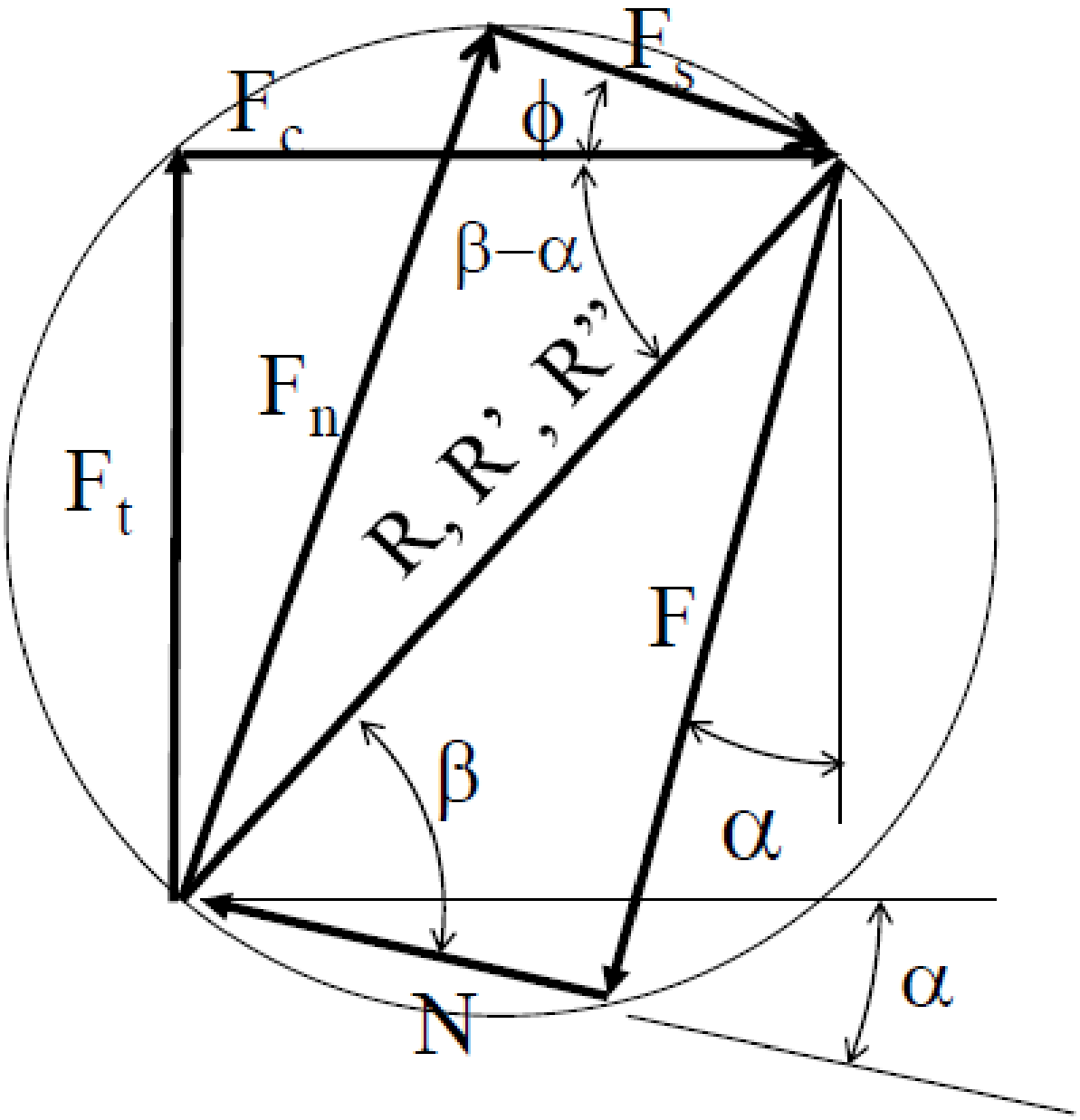


F: Friction Force
N: Normal Force

F_s: Shear Force
F_n: Normal Force

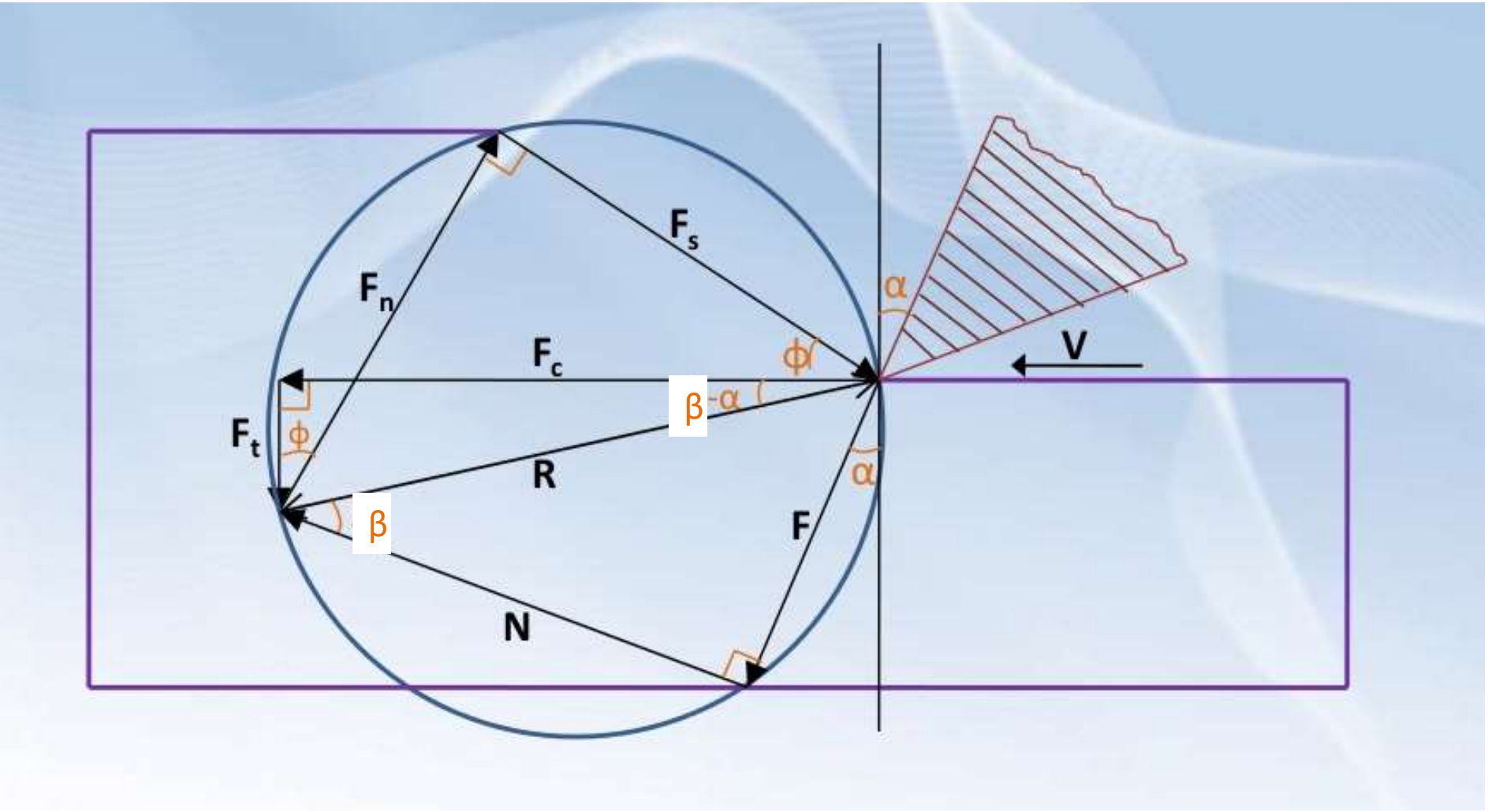
F_c: Cutting Force
F_t: Thrust Force

Merchant's circle Diagram



Merchant's circle Diagram

Force Diagram



Merchant's circle Diagram

Force Diagram

- ***How to measure Φ (dependent) variable, since α is independent variables?????***

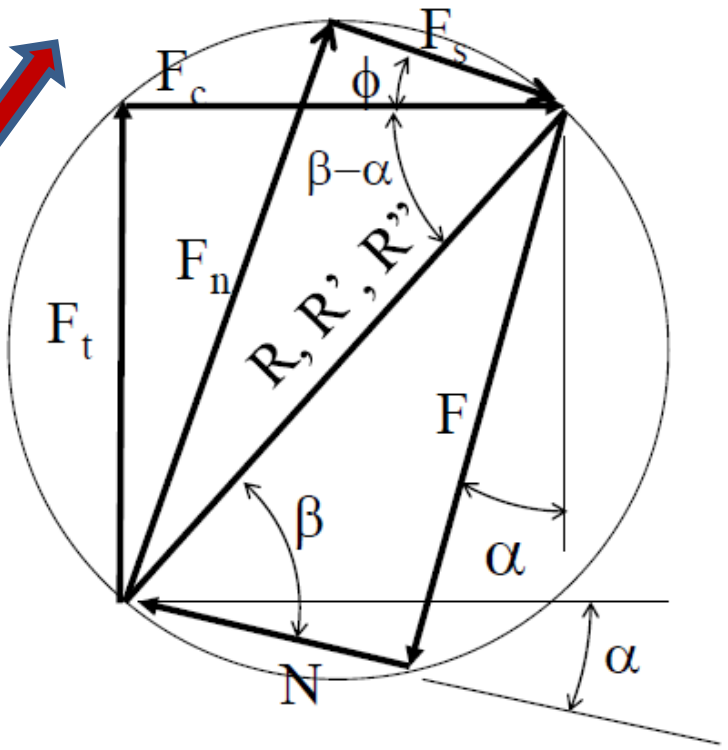
- Forces acting on the tool that can not be measured:

$$F_s = R\cos(\phi+\beta-\alpha) = F_c\cos\phi - F_t\sin\phi$$

$$F_n = R\sin(\phi+\beta-\alpha) = F_c\sin\phi + F_t\cos\phi$$

$$F = R\sin\beta = F_c\sin\alpha + F_t\cos\alpha$$

$$N = R\cos\beta = F_c\cos\alpha - F_t\sin\alpha$$



- Forces acting on the tool that can be measured

$$F_c = R\cos(\beta-\alpha) = \frac{F_s\cos(\beta-\alpha)}{\cos(\Phi+\beta-\alpha)}$$

$$F_t = R\sin(\beta-\alpha) = \frac{F_s\sin(\beta-\alpha)}{\cos(\Phi+\beta-\alpha)}$$

- ***How theses forces can be measured?????***

Forces in Metal Cutting

- Equations can be derived to relate the forces that cannot be measured to the forces that can be measured:

$$F = F_c \sin\alpha + F_t \cos\alpha$$

$$N = F_c \cos\alpha - F_t \sin\alpha$$

$$F_s = F_c \cos\phi - F_t \sin\phi$$

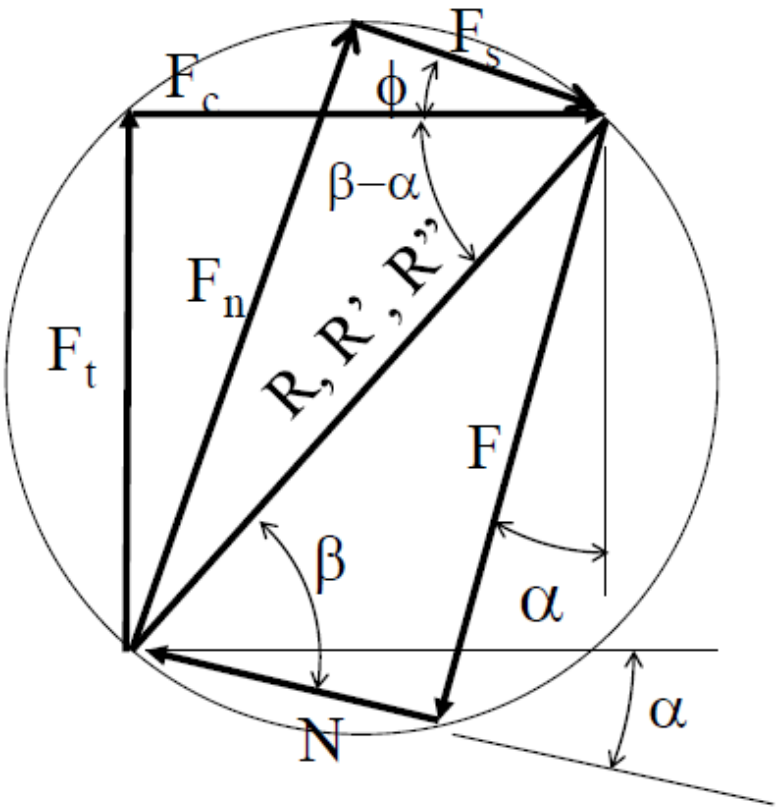
$$F_n = F_c \sin\phi + F_t \cos\phi$$

- Based on these calculated force, shear stress and coefficient of friction can be determined

Coefficient of Friction

- Friction angle:

$$\tan\beta = \frac{F}{N}$$



- Coefficient of friction between tool and chip:

$$\mu = \frac{F}{N}$$

- Coefficient of friction related to friction angle as follows:

$$\mu = \tan\beta \Rightarrow \mu = \tan\beta = \frac{F_t + F_c \tan\alpha}{F_c - F_t \tan\alpha}.$$

Coefficient of Friction

- In metal cutting, μ generally ranges *from about 0.5 to 2*, **thus indicating** that the chip **encounters considerable frictional resistance while climbing up the rake face of the tool**

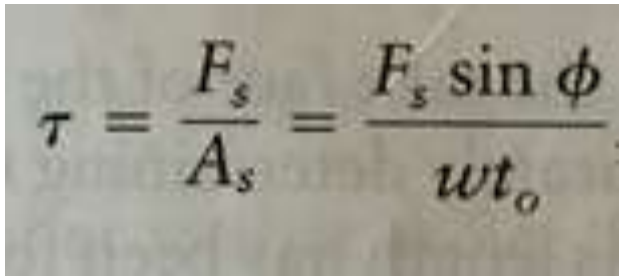
Shear Stress

- The forces in the shear plane can be resolved into ***shear*** and ***normal*** forces
  - The area, A_o, of the shear plane is:

$$A_s = \frac{t_o w}{\sin\phi}$$

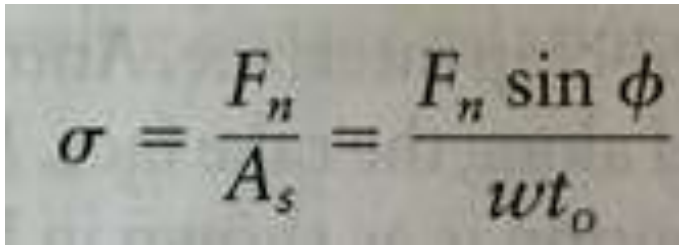
- Shear stress acting along the shear plane:

$$\tau = S = \frac{F_s}{A_s}$$



Shear stress = shear strength of work material during cutting

- And the normal stress, σ, is:



Advantages of Merchant's circle

Proper use of MCD enables the followings :-

- **Easy, quick and reasonably accurate determination of several other forces from a few forces involved in machining.**
- **Friction at chip-tool interface and dynamic yield shear strength can be easily determined.**
- **Equations relating the different forces are easily developed.**

Disvantages of Merchant's circle

Some limitations of use of MCD are :-

- **Merchant's Circle Diagram (MCD) is valid only for orthogonal cutting.**
- **By the ratio, F/N, the MCD gives apparent (not actual) coefficient of friction.**
- **It is based on single shear plane theory.**

Shear angle relation ships

How to measure Φ???

❑ The Merchant Equation

❑ This theory based on the assumption that

- The shear angle adjusts itself so that the cutting force is a minimum [*the minimum cutting force required when the shear plane takes place to have minimum cutting energy*].
- Of all the possible angles at which shear deformation can occur, the work material will select a shear plane angle ϕ that minimizes energy, given by:

$$\phi = 45 + \frac{\alpha}{2} - \frac{\beta}{2}$$

→ In degree

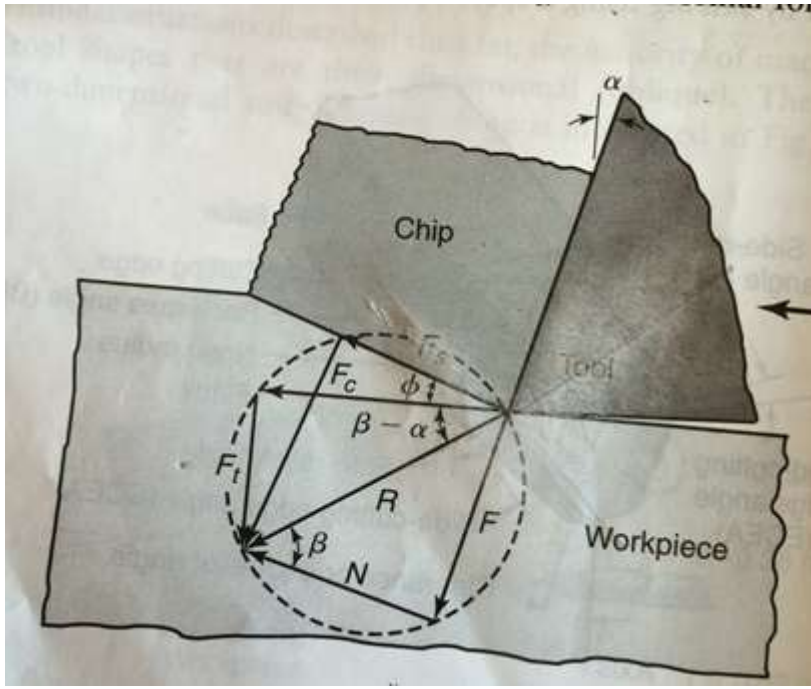
$$\phi = 0.785 + \frac{\alpha}{2} - \frac{\beta}{2}$$

→ In rad

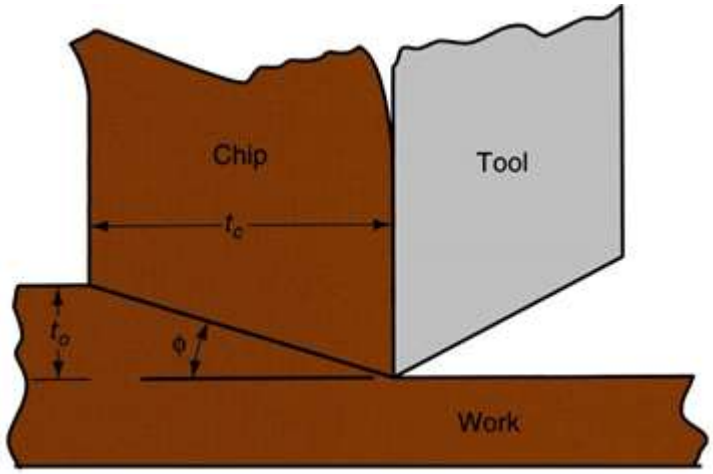
What the Merchant Equation Tells Us

- Lower shear plane angle

$$\phi = 45 + \frac{\alpha}{2} - \frac{\beta}{2}$$



- As *the rank angle decreases* (the friction at the tool-chip interface increases and thus **β** increases) *the shear angle decreases*, and thus the chip becomes thicker.
- This result is expected, because **decreasing α** and **increasing β** tend to present greater resistance to chip as it moves up the rake face of the tool, ***thus making the chip thicker, indicating a lower shear angle***

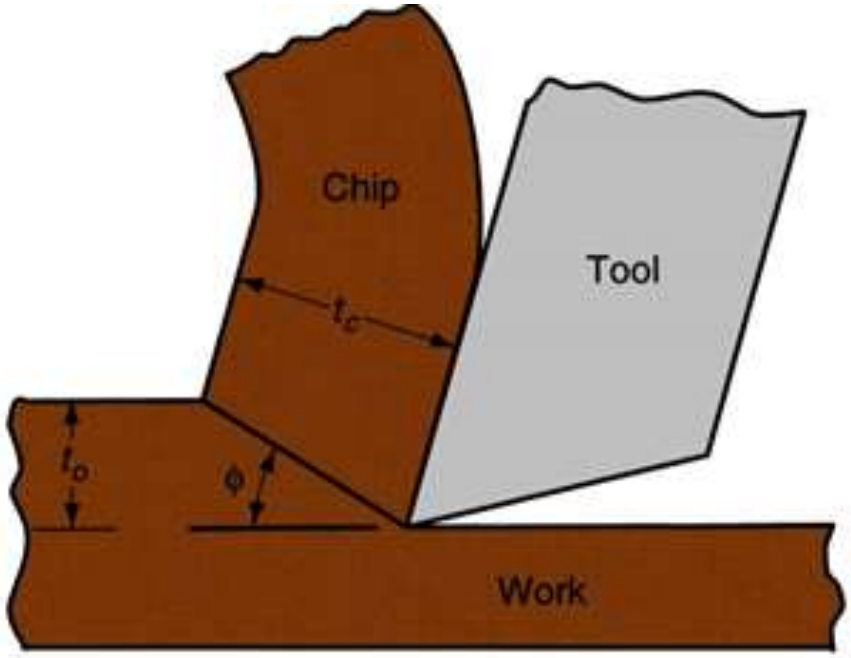


What the Merchant Equation Tells Us

❑ Higher shear plane angle

$$\phi = 45 + \frac{\alpha}{2} - \frac{\beta}{2}$$

- To increase shear plane angle
  - Increase the rake angle
  - Reduce the friction angle (or coefficient of friction) $\mu = \tan\beta$



Effect of Higher Shear Plane Angle

$$\phi = 45 + \frac{\alpha}{2} - \frac{\beta}{2}$$

- Higher shear plane angle means smaller shear plane which means small chip thickness, lower shear force, cutting forces, power, and temperature

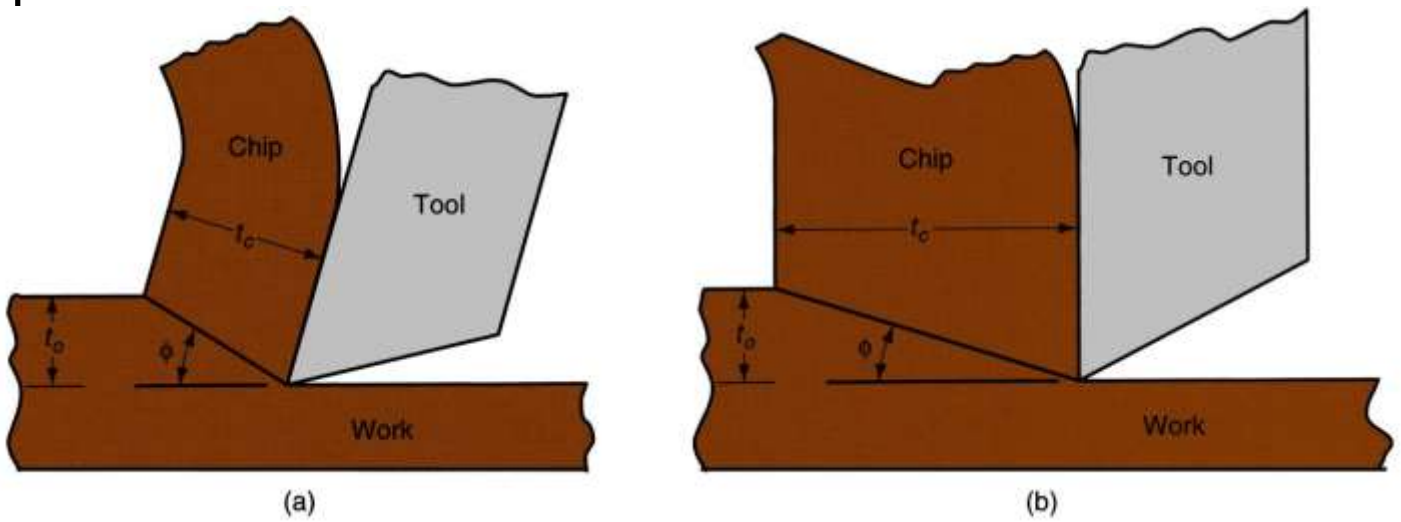
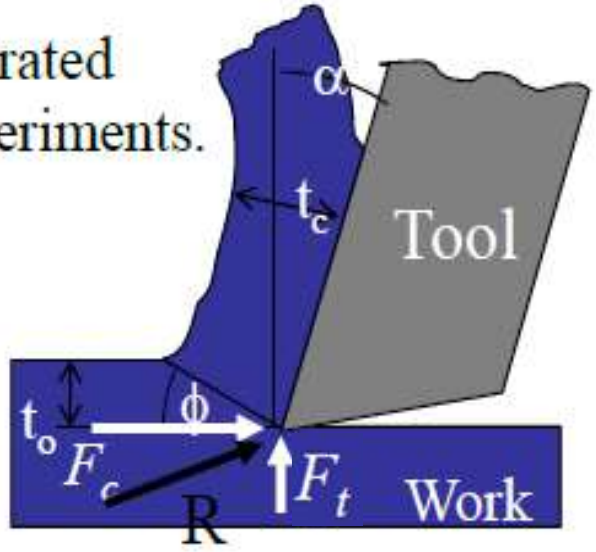


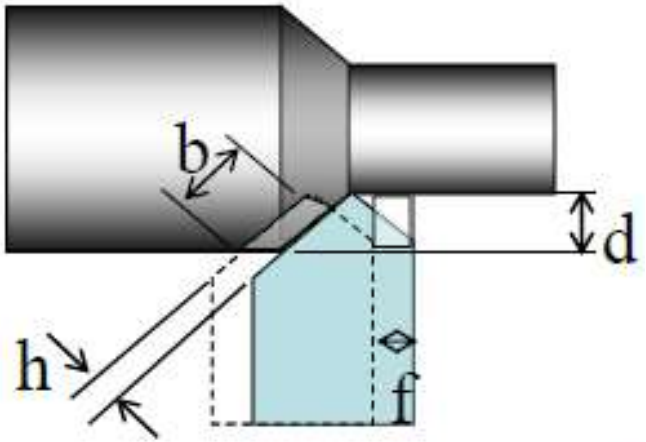
Figure (a) & (b): Effect of shear plane angle ϕ: (a) higher ϕ with a resulting lower shear plane area; (b) smaller ϕ with a corresponding larger shear plane area. Note that the rake angle is larger in (a), which tends to increase shear angle according to the Merchant equation

How to measure F_c & F_t

- Cutting Force: $F_c = bh\tau_s\left[\dfrac{\cos(\beta-\alpha)}{\sin\phi\cos(\phi+\beta-\alpha)}\right] = K_c bh \qquad K_c\left[N/mm^2\right]$
- Thrust Force: $F_t = bh\tau_s\left[\sin\dfrac{\cos(\beta-\alpha)}{\sin\phi\cos(\phi+\beta-\alpha)}\right] = K_t bh$

K_c and K_t must be calibrated through machining experiments.





Top View

Specific energy

- Total specific energy (*the total energy per unit volume of material removed*) is dissipated into ***friction*** and ***shearing***:

$$U_t = U_f + U_s$$

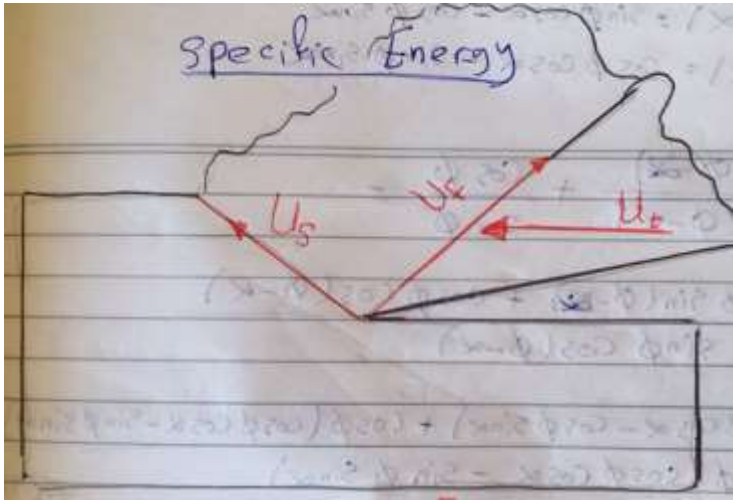
❑ Specific energy = $\frac{Energy\ (J)}{Volume\ (m^3.cm^3,mm^3)}$

❑ Specific energy = $\frac{Power\ (\frac{J}{s})}{Volume\ Rate\ (\frac{m^3}{s},\frac{mm^3}{s},\frac{cm^3}{s})}$

❑ Volume rate of removed material = t_o *(mm)* * W *(mm)* * V *(mm/s)*

- Volume rate is constant.
- Where V is the cutting velocity.

❑ Power = (*Force* * *Velocity*)



Power and Energy Relationships

- A machining operation requires power
- The total power to perform machining can be computed from:

$$P_c = F_c\, v$$

where

- ✓ P_c = cutting power;
- ✓ F_c = cutting force;
- ✓ v = cutting speed or cutting velocity.

Power and Energy Relationships

- Total specific energy is

$$U_t = \frac{Total\ power}{Volume\ rate} = \frac{P_C}{R_{MRR}} = \frac{F_C V}{V\ to\ W} = \frac{F_c}{t_o W}$$

$$U_F = \frac{power\ for\ friction}{Volume\ rate} = \frac{P_F}{R_{MRR}} = \frac{F\ Vc}{V\ t_o W}$$

V_c : chip velocity

$$U_S = \frac{power\ for\ shearing}{Volume\ rate} = \frac{P_s}{R_{MRR}} = \frac{F_s Vs}{V\ to W}$$

V_s : shear velocity

- Units for specific energy are typically N-m/mm^3 or (lb-in/in^3) J/mm^3
- Unit for U is generally J/mm3.

Approximate Specific Energy Requirements In Machining Operations

Material	Specific energy* W-s/mm^3
Aluminum alloys	0.4–1.1
Cast irons	1.6–5.5
Copper alloys	1.4–3.3
High-temperature alloys	3.3–8.5
Magnesium alloys	0.4–0.6
Nickel alloys	4.9–6.8
Refractory alloys	3.8–9.6
Stainless steels	3.0–5.2
Steels	2.7–9.3
Titanium alloys	3.0–4.1

*At drive motor, corrected for 80% efficiency; multiply the energy by 1.25 for dull tools.

Cutting Temperature

- Approximately 98% of the energy in machining is converted into heat
- This can cause temperatures to be very high at the tool-chip
- The remaining energy (about 2%) is retained as elastic energy in the chip

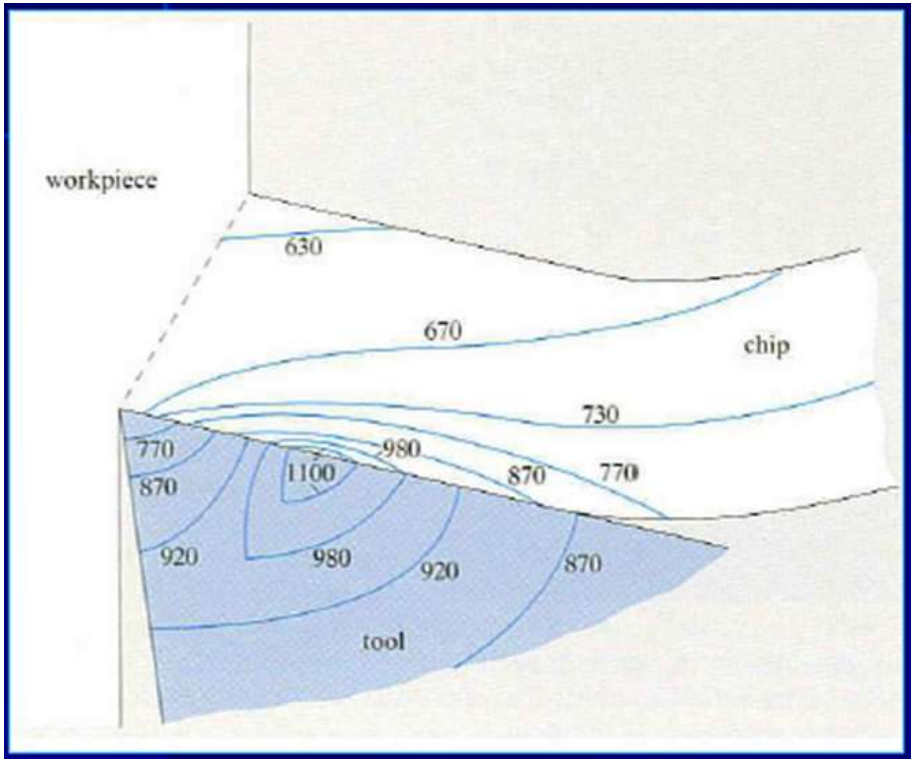
❑ The main source of heat generation.

- **Because of the work done *in shearing*** and ***in overcoming friction on the rake face of the tool***, the main sources of heat generation are :

1. Plastic deformation in the shearing process and
2. Friction on the tool-chip interface.

❑ If the tool is dull or worn, heat is also generated by the tool tip rubbing against the machine surface.

Temperature in metal cutting



Temperature gradient (K) in the cutting zone when machining steel.

- A significant ***temperature rise*** is due to ***large plastic strain*** and ***very high strain rate*** although the process is normally carried out at ambient temperature.
- Strain rate is high in cutting and almost all the ***plastic work*** is converted into ***heat***.
- Very ***high temperature*** is created in the ***secondary deformation zone***.
- At very ***high strain rate*** → no time for heat dissipation → temperature rise.

Temperature in metal cutting is therefore an important factor affecting the choice of tool materials, tool life, type of lubricant,

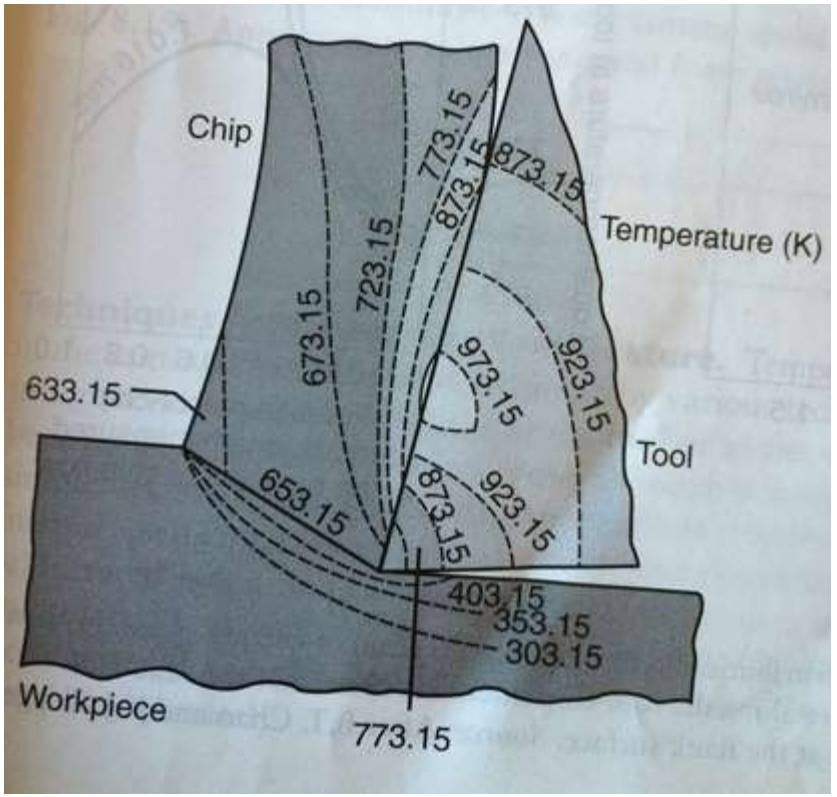
Cutting Temperature

- The energy dissipated in machining operations is converted into heat, which in turn, raises temperature cutting. Temperature rise in cutting is important because :

1. **Adversely affect strength, hardness, and wear resistance of cutting tool.**
2. **Cause dimensional changes in the part being machined, making control of dimensional accuracy difficult.**
3. **Can induce thermal damage to the machined surface, adversely affecting its properties and service life.**
4. **The machine tool itself may be subjected to temperature gradients, causing distortion of the tool and adversely affecting dimensional control.**

Cutting Temperature

- Various studies have been made of temperature of cutting.
- The figure shows ***that there are severe temperature gradients in the cutting zone***.
- An approximate expression for calculating ***the mean temperature for the orthogonal cutting is derived by Nathan Cook*** from dimensional analysis using experimental data for various work materials



$$T = \frac{0.4U}{\rho C}\left(\frac{vt_o}{K}\right)^{0.333}$$

where T = temperature rise at tool-chip interface; U = specific energy; v = cutting speed; t_o = chip thickness before cut; ρC = volumetric specific heat of work material; K = thermal diffusivity of work material